Curvature-constrained multi-goal trajectory planning

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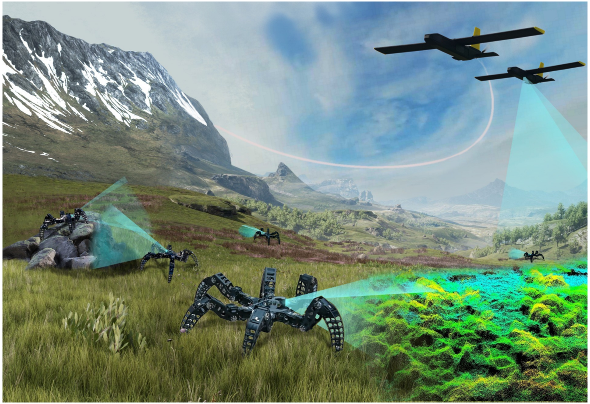
Machine Learning and Modelling Seminar (SUI)

Computational Robotics Laboratory – Artificial Intelligence Center

- **Robotic information gathering** – build phenomena model using measurements collected by mobile robots
- **Quality guarantee** of the found solution (**tight lower bound**) and **computationally efficient** solutions
- **Machine learning** in online and **lifelong learning** scenarios (improving estimations/computational performance)



Computational Robotics Laboratory (CRL)
https://comrob.fel.cvut.cz

- 10 phd students
- 2 technicians
- 15+ undergraduates

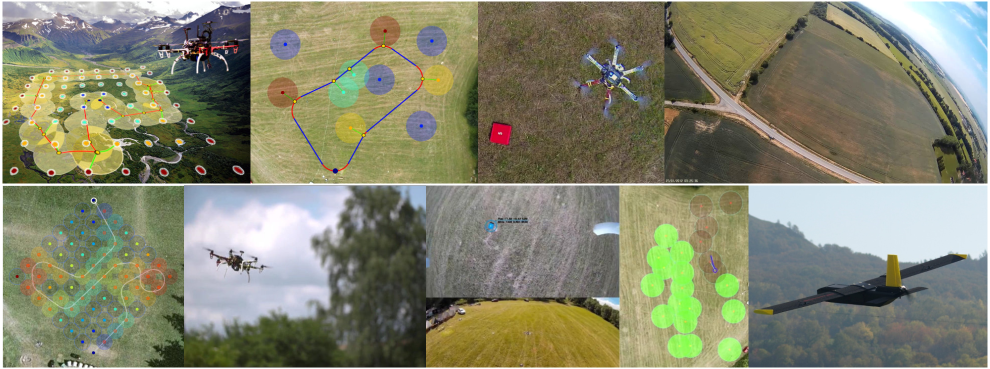
Overview of the Lecture

- Multi-goal planning with curvature-constrained trajectories
- Dubins Vehicle Model
- Dubins Touring Problem (DTP)
- Dubins Traveling Salesman Problem (DTSP)
- Dubins Traveling Salesman Problem with Neighborhoods (DTSPN)
- Generalizations of the Dubins Vehicle Model
  - Variable speed trajectories
  - 3D Trajectories

Outline

Motivation – Surveillance Missions with Aerial Vehicles

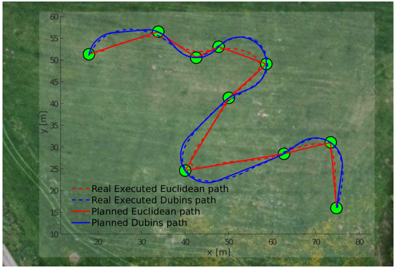
- Find **curvature-constrained** path/trajectory to collect the most valuable measurements with shortest possible path/time or under limited travel budget.

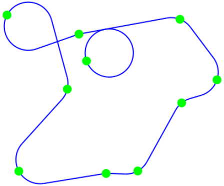


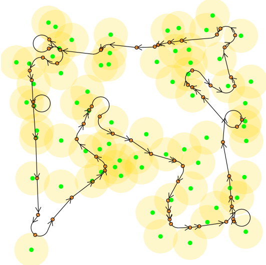
- We need curvature-constrained trajectory to fit **motion constraints** of the vehicles.
- Limited turning radius needs to **reason on how the vehicle approaches the goal locations**.
- For routing problems limited travel budget, we need **realistic estimation of the travel cost**.

Challenges of Multi-goal Curvature-constrained Path/Trajectory Planning

- The problem of visiting a set of targets is **multi-goal planning** problem that includes **combinatorial optimization** to find the best sequence of visits to the targets.
  - Routing problem formulated as the **Traveling Salesman Problem** (**TSP**).
  - The quality of the sequence needs to be evaluted as a cost of the **multi-goal** path/trajectory.
  - The multi-goal path/trajectory needs to respect motion constraints, such as limited turning radius.







- A high number of possible sequences is expected to be examined in route planning, therefore, the evaluation of the **sequence quality** should be *computationally efficient* (the multi-goal trajectory planning).
- We can also ask for **tight lower bounds** on the optimal solution value of the multi-goal trajectory.

Outline

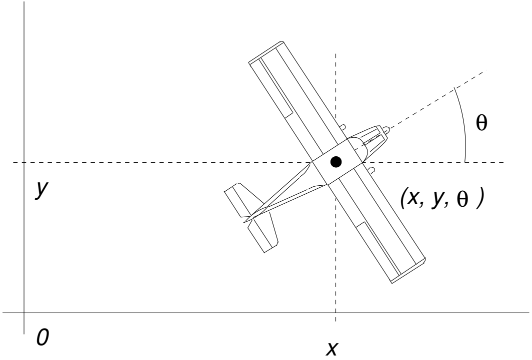
Dubins Vehicle

- Non-holonomic vehicle such as car-like or aircraft can be modeled as the Dubins vehicle:
  - Constant forward velocity;
  - Limited minimal turning radius ρ;
  - Vehicle state is represented by a triplet $q = (x, y, \theta)$, where
  - Position is $(x, y) \in \mathbb{R}^2$, vehicle heading is $\theta \in \mathbb{S}^2$, and thus $q \in SE(2)$.

The vehicle motion can be described by the equation

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos\theta \\ \sin\theta \\ \frac{u}{\rho} \end{bmatrix}, \quad |u| \le 1,$$

where u is the control input.

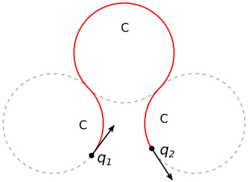


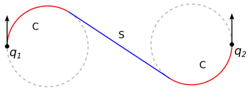
Optimal (Point-to-Point) Maneuvers for Dubins Vehicle

- For two states $q_1 \in SE(2)$ and $q_2 \in SE(2)$ in the environment without obstacles $\mathcal{W} = \mathbb{R}^2$, the optimal path connecting q_1 with q_2 can be characterized as one of two main types
  - **CCC** type: LRL, RLR;
  - **CSC** type: LSL, LSR, RSL, RSR;

where S – straight line arc, C – circular arc oriented to left (L) or right (R).

L. E. Dubins (1957) – American Journal of Mathematics





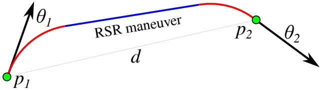
Markov-Dubins Path – A.A.Markov (1887) optimal path problem solved by Dubins using a number of constructive geometric arguments. It can also be solved by the Pontryagin's maximum principle.

- The optimal paths are called **Dubins maneuvers**.
  - Constant velocity: $v(t) = v$ and turning radius ρ (path vs. trajectory).
  - **Six** types of trajectories connecting any configuration in $SE(2)$. *(Without obstacles)*
  - The control u is according to C and S type one of three possible values $u \in \{-1, 0, 1\}$.
- Headings are not prescribed and the routing problems are formulated as
  - **Dubins Traveling Salesman Problem (with Neighborhoods)** (**DTSP**(**N**));
  - **Dubins Orienteering Problem (with Neighborhoods)**.

Challenges of Dubins Vehicle in Solving Dubins Routing Problems

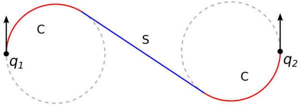
- For the minimal turning radius ρ, the **optimal path** connecting $\boldsymbol{q}_1 \in SE(2)$ and $\boldsymbol{q}_2 \in SE(2)$ can be found analytically.

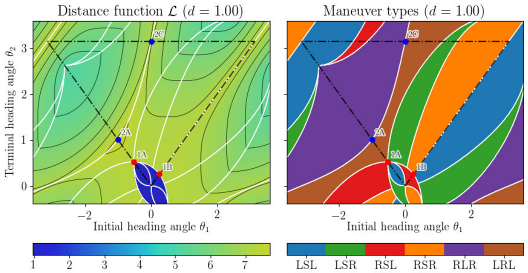
  L. E. Dubins (1957) – American Journal of Mathematics

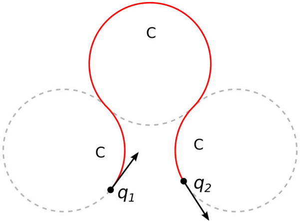


- Two types of optimal Dubins maneuvers: CSC and CCC.
- The length of the optimal maneuver $\mathcal{L}$ has a closed-form solution.
  - It is **piecewise-continuous function**;
  - (continuous for $\|(\boldsymbol{p}_1, \boldsymbol{p}_2)\| > 4\rho$).

Can be computed in less than 0.5 μs.







Dubins Traveling Salesman Problem (DTSP)

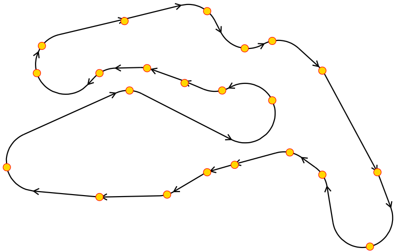
- Determine (closed) shortest Dubins path visiting each $\boldsymbol{p}_i \in \mathbb{R}^2$ of the given set of n locations $P = \{\boldsymbol{p}_1, \ldots, \boldsymbol{p}_n\}$.

1. Permutation $\Sigma = (\sigma_1, \ldots, \sigma_n)$ of visits (sequencing).

   Combinatorial optimization

2. *Headings* $\Theta = \{\theta_{\sigma_1}, \theta_{\sigma_2}, \ldots, \theta_{\sigma_n}\}$, $\theta_i \in [0, 2\pi)$, *for* $\boldsymbol{p}_{\sigma_i} \in P$.

   Continuous optimization

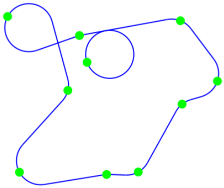


- **DTSP** is an optimization problem over all possible **sequences** Σ and **headings** Θ at the states $(\boldsymbol{q}_{\sigma_1}, \boldsymbol{q}_{\sigma_2}, \ldots, \boldsymbol{q}_{\sigma_n})$ such that $\boldsymbol{q}_{\sigma_i} = (\boldsymbol{p}_{\sigma_i}, \theta_{\sigma_i})$, $\boldsymbol{p}_{\sigma_i} \in P$

$$\text{minimize}_{\Sigma,\Theta} \quad \sum_{i=1}^{n-1} \mathcal{L}(\boldsymbol{q}_{\sigma_i}, \boldsymbol{q}_{\sigma_{i+1}}) + \mathcal{L}(\boldsymbol{q}_{\sigma_n}, \boldsymbol{q}_{\sigma_1})$$

$$\text{subject to} \quad \boldsymbol{q}_i = (\boldsymbol{p}_i, \theta_i) \; i = 1, \ldots, n,$$

where $\mathcal{L}(\boldsymbol{q}_{\sigma_i}, \boldsymbol{q}_{\sigma_j})$ is the length of Dubins path between $\boldsymbol{q}_{\sigma_i}$ and $\boldsymbol{q}_{\sigma_j}$.

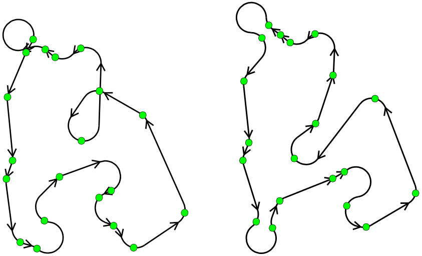


The continuous domain of the heading angles is similar to the regions in the TSPN-like problem formulations.

Approaches to the Dubins Traveling Salesman Problem

- The key difficulty of the DTSP is that the trajectory cost depends on
  - Order of the visits to the locations;
  - Headings at the goal locations.

We need the sequence to determine headings, but headings may influence the sequence.

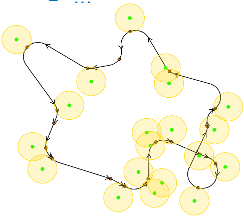


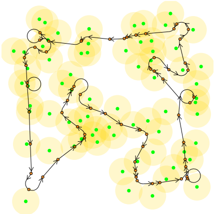
- The Dubins TSP is **sequence-dependent problem.**
- Two fundamental approaches can be used.
  - **Decoupled** approach
    1. Find sequence, e.g., using the Euclidean TSP.
    2. Solve the continuous n-variables optimization for determining optimal headings, e.g., using discretization. The problem is called **Dubins Touring Problem** (**DTP**).
  - **Transformation approach** (**sampling-based**) approach – sampling headings into discrete sets of values and transforming the problem to a variant of the **Generalized TSP**.

Purely combinatorial optimization.

Existing Approaches to the DTSP(N)

- Heuristic (decoupled & evolutionary) approaches
  - *Savla et al., 2005*
  - *Ma and Castanon, 2006*
  - *Macharet et al., 2011*
  - *Macharet et al., 2012*
  - *Ny et al., 2012*
  - *Yu and Hang, 2012*
  - *Macharet et al., 2013*
  - *Zhant et al., 2014*
  - *Macharet and Campost, 2014*
  - *Váňa and Faigl, 2015*
  - *Isaiah and Shima, 2015*
  - ...





- Sampling-based approaches
  - *Obermeyer, 2009*
  - *Oberlin et al., 2010*
  - *Macharet et al., 2016*
- Decoupled approaches *For a given sequence*
  - Convex optimization
    - (Only if the locations are far enough)
    - *Goac et al., 2013*
  - **Lower bound for the DTSP**
    - Dubins Interval Problem (DIP)
    - *Manyam et al., 2016*
    - DIP-based inform sampling
    - *Váňa and Faigl, 2017*
  - **Lower bound for the DTSPN**
    - Using Generalized DIP (GDIP)
    - *Váňa and Faigl, 2018, 2020*

Outline

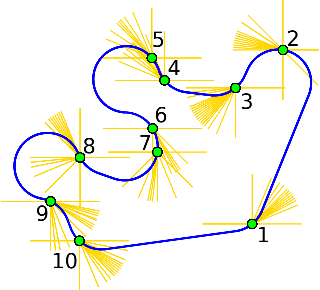
Dubins Touring Problem – DTP

- For a sequence of the n waypoint locations $P = (p_1, \ldots p_n)$, $p_i \in \mathbb{R}^2$, the **Dubins Touring Problem** (**DTP**) stands to determine the **optimal headings** $T = \{\theta_1, \ldots, \theta_n\}$ at the waypoints q_i such that

$$\text{minimize}_T \quad \mathcal{L}(T, P) = \sum_{i=1}^{n-1} \mathcal{L}(q_i, q_{i+1}) + \mathcal{L}(q_n, q_1)$$

$$\text{subject to} \quad q_i = (p_i, \theta_i), \ \ \theta_i \in [0, 2\pi), \ \ p_i \in P,$$

where $\mathcal{L}(q_i, q_j)$ is the length of the Dubins maneuver connecting q_i with q_j.



- The DTP is a continuous optimization problem.
- The term $\mathcal{L}(q_n, q_1)$ is for the closed tour, e.g., for the Dubins TSP.

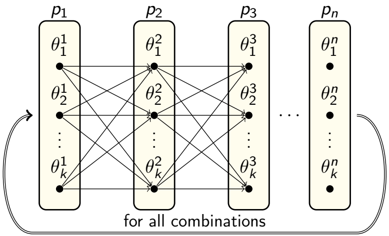
The DTP can also be utilized for open paths such as the OP with Dubins vehicle.

Sampling-based Solution of the DTP

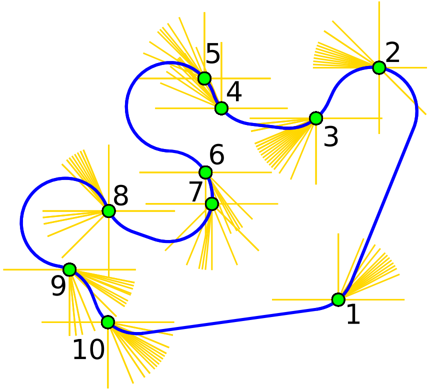
- For a closed sequence of the waypoint locations

$$P = (p_1, \ldots, p_n).$$

- We can sample possible heading values at each location i into a discrete set of k headings, i.e., $\Theta^i = \{\theta^i_1, \ldots, \theta^i_k\}$ and create a graph of all possible Dubins maneuvers.



- For a set of heading samples, the optimal solution can be found by a forward search of the graph in $O(nk^3)$.



For open sequence we do not need to evalute all possible initial headings, and the complexity is $O(nk^2)$.

- **The problem is to determine the most suitable heading samples.**

Example of Heading Sampling – Uniform vs. Informed

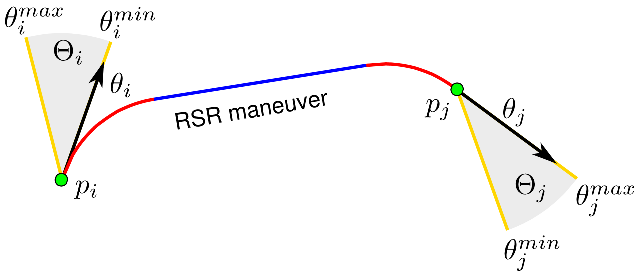
$N = 224,\ T_{cpu} = 128$ ms
$\mathcal{L} = 19.8,\ \mathcal{L}_U = 13.8$

$N = 128,\ T_{cpu} = 76$ ms
$\mathcal{L} = 14.4,\ \mathcal{L}_U = 14.2.$

- N is the total number of samples, i.e., 32 samples per waypoint for uniform sampling.
- $\mathcal{L}$ is the length of the tour (blue) and $\mathcal{L}_U$ is the lower bound (red) determined as a solution of the **Dubins Interval Problem** (**DIP**).

Dubins Interval Problem (DIP)

- **Dubins Interval Problem (DIP)** is a generalization of Dubins maneuvers to the shortest path connecting two points p_i and p_j.
- In the DIP, the leaving interval Θ_i at p_i and the arrival interval Θ_j at p_j are consider (not a single heading value).
- The optimal solution can be found analytically. *Manyam et al. (2015)*

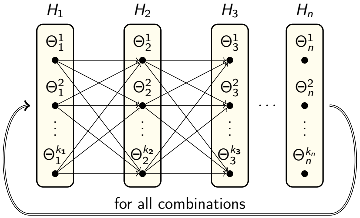


- Solution of the DIP is a tight lower bound for the DTP.
- Solution of the DIP is not a feasible solution of the DTP.

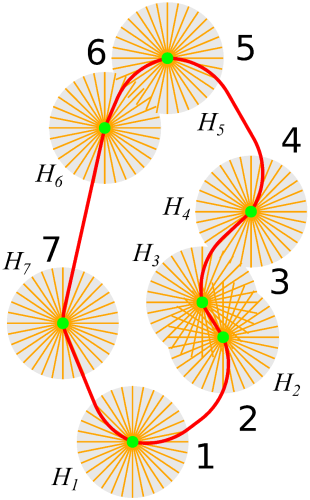
Notice, for $\Theta_i = \Theta_j = \langle 0, 2\pi)$ the optimal maneuver for DIP is a straight line segment.

Lower Bound of the DTP

- For a discrete set of heading intervals $\mathcal{H} = \{H_1, \ldots, H_n\}$, where $H_i = \{\Theta_i^1, \Theta_i^2, \ldots, \Theta_i^{k_i}\}$, a similar graph as for the DTP can be constructed with the edge cost determined by the solution of the associated DIP.



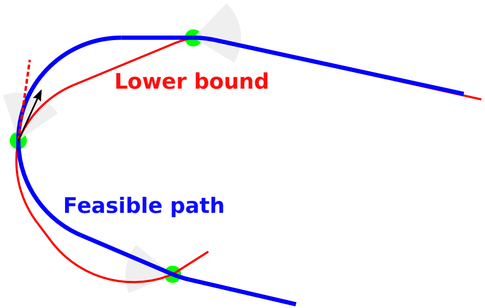
- The forward search of the graph with dense samples provides a tight lower bound of the DTP. *Manyam and Rathinam, 2015*



Lower Bound and Feasible Solution of the DTP

- The arrival and departure angles may not be the same.

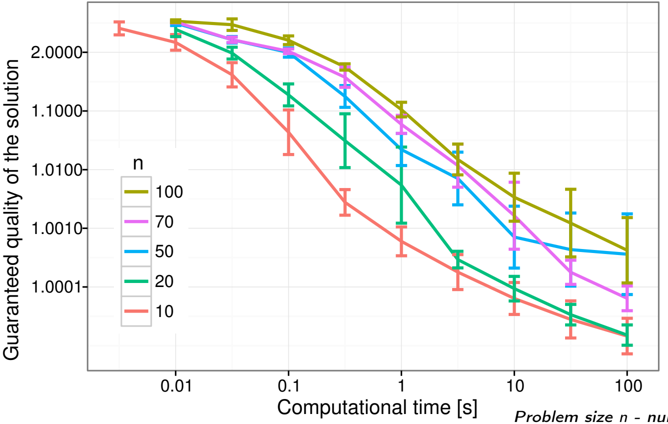
The lower bound solution is not a feasible solution of the DTP.



- DTP solution – use any particular heading of each interval in the lower bound solution.

The DIP-based Sampling of Headings in the DTP

- Using heading intervals for a sequence of waypoints and a solution of the DIP, we can determine **lower bound** of the DTP using the forward search graph as for the DTP.
- **Relative optimality gap** – the ratio of the lower bound and feasible solution of the DTP.



Problem size n - number of target locations.

Iteratively-Refined Informed Sampling (IRIS) of Headings in the DTP

- Iterative refinement of the heading intervals $\mathcal{H}$ up to the angular resolution ϵ_{req}.
- The angular resolution is gradually decreased for the most promising intervals.
- `refineDTP` – divide the intervals of the lower bound solution.
- `solveDTP` – solve DTP using the heading from the refined intervals.
- It simultaneously provides **feasible** and **lower bound** solutions of the DTP.

  The lower bound provides a tight estimation of the solution quality.

Algorithm 1: Iteratively-Refined Informed Sampling for the DTP

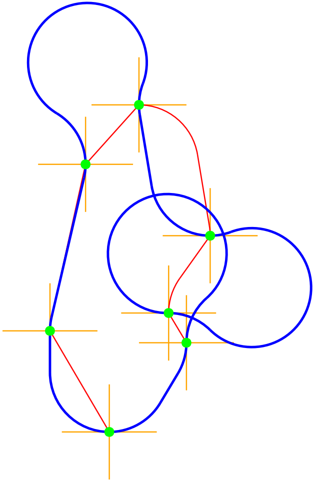
Input: P – Target locations to be visited
Input: ϵ_{req} – Requested angular resolution
Input: α_{req} – Requested quality of the solution
Output: T – A tour visiting the targets

$\epsilon \leftarrow 2\pi$ // initial angular resolution;
$\mathcal{H} \leftarrow$ `createIntervals`(P, ϵ) // initial intervals;
$\mathcal{L}_L \leftarrow 0$ // init lower bound;
$\mathcal{L}_U \leftarrow \infty$ // init upper bound;
while $\epsilon > \epsilon_{req}$ and $\mathcal{L}_U/\mathcal{L}_L > \alpha_{req}$ **do**
  $\epsilon \leftarrow \epsilon/2$;
  $(\mathcal{H}, \mathcal{L}_L) \leftarrow$ `refineDTP`$(\mathcal{P}, \epsilon, \mathcal{H})$;
  $(T, \mathcal{L}_U) \leftarrow$ `solveDTP`$(\mathcal{P}, \mathcal{H})$;
end
return T;

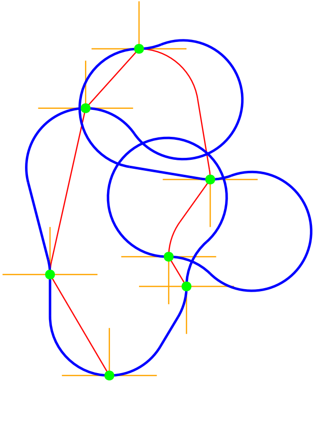
- The first solution is provided very quickly – **any-time algorithm**.

Faigl, J., Váňa, P., Saska, M., Báča, T., and Spurný, V.: *On solution of the Dubins touring problem*, ECMR, 2017.

Uniform vs Informed Sampling

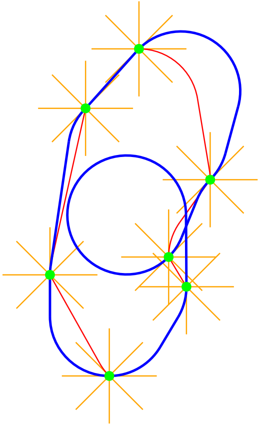


$\epsilon = 2\pi/4$, $N = 28$, $T_{CPU} = 8$ ms
$\mathcal{L} = 27.9$, $\mathcal{L}_U = 13.2$

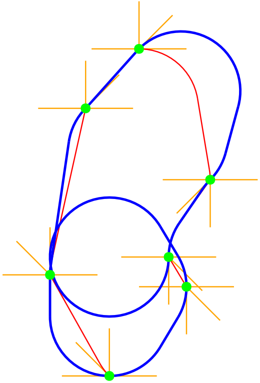


$\epsilon = 2\pi/4$, $N = 21$, $T_{CPU} = 8$ ms
$\mathcal{L} = 29.9$, $\mathcal{L}_U = 13.2$

Uniform vs Informed Sampling

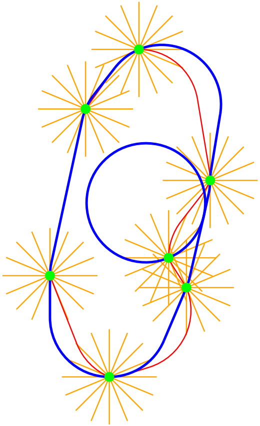


$\epsilon = 2\pi/8$, $N = 56$, $T_{CPU} = 16$ ms
$\mathcal{L} = 20.8$, $\mathcal{L}_U = 13.2$

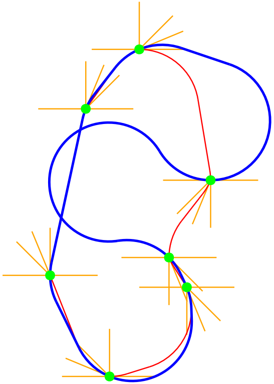


$\epsilon = 2\pi/8$, $N = 28$, $T_{CPU} = 20$ ms
$\mathcal{L} = 21.0$, $\mathcal{L}_U = 13.2$

Uniform vs Informed Sampling

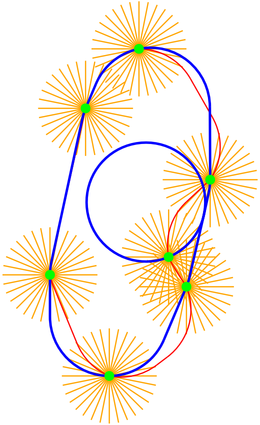


$\epsilon = 2\pi/16$, $N = 112$, T_{CPU}= 40 ms
$\mathcal{L} = 20.3$, $\mathcal{L}_U = 13.5$

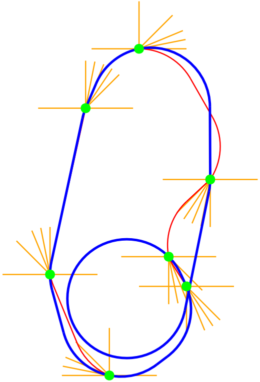


$\epsilon = 2\pi/16$, $N = 35$, T_{CPU}= 24 ms
$\mathcal{L} = 20.1$, $\mathcal{L}_U = 13.5$

Uniform vs Informed Sampling

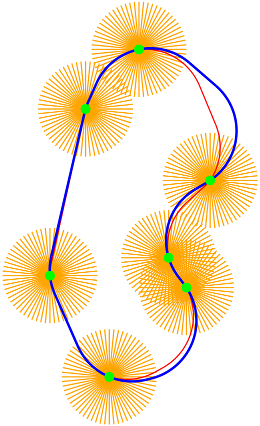


$\epsilon = 2\pi/32$, $N = 224$, $T_{CPU} = 140$ ms
$\mathcal{L} = 19.8$, $\mathcal{L}_U = 13.8$

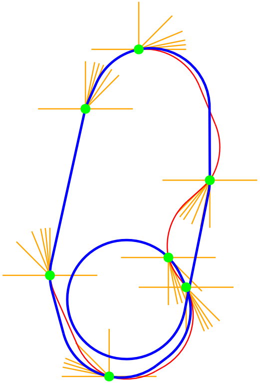


$\epsilon = 2\pi/32$, $N = 44$, $T_{CPU} = 32$ ms
$\mathcal{L} = 19.9$, $\mathcal{L}_U = 13.8$

Uniform vs Informed Sampling

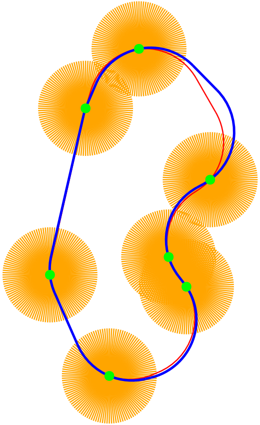


$\epsilon = 2\pi/64$, $N = 448$, T_{CPU} = 456 ms
$\mathcal{L} = 14.5$, $\mathcal{L}_U = 14.5$

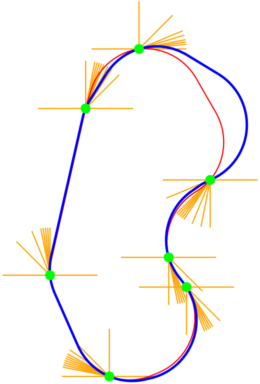


$\epsilon = 2\pi/64$, $N = 51$, T_{CPU} = 48 ms
$\mathcal{L} = 19.9$, $\mathcal{L}_U = 13.9$

Uniform vs Informed Sampling

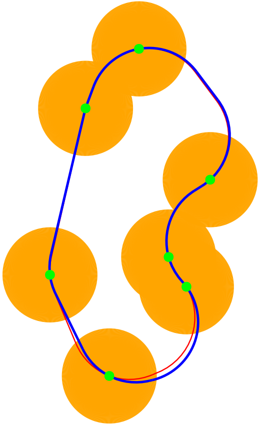


$\epsilon = 2\pi/128$, $N = 896$, $T_{\text{CPU}} = 1620$ ms
$\mathcal{L} = 14.5$, $\mathcal{L}_U = 14.5$

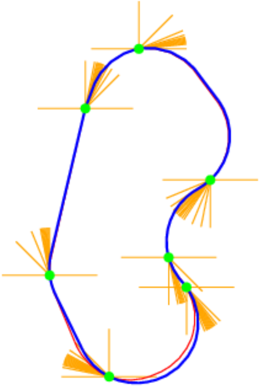


$\epsilon = 2\pi/128$, $N = 70$, $T_{\text{CPU}} = 60$ ms
$\mathcal{L} = 14.8$, $\mathcal{L}_U = 14.1$

Uniform vs Informed Sampling



$\epsilon = 2\pi/256$, $N = 1792$, $T_{CPU} = 6784$ ms
$\mathcal{L} = 14.4$, $\mathcal{L}_U = 14.3$



$\epsilon = 2\pi/256$, $N = 100$, $T_{CPU} = 88$ ms
$\mathcal{L} = 14.4$, $\mathcal{L}_U = 14.3$

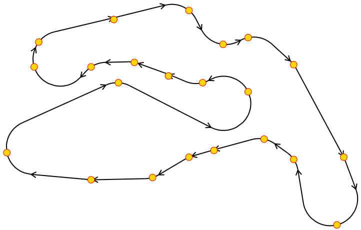
Outline

Dubins Traveling Salesman Problem (DTSP)

- An optimization over all possible **sequences** Σ and **headings** Θ.

$$\text{minimize}_{\Sigma,\Theta} \quad \sum_{i=1}^{n-1} \mathcal{L}(\boldsymbol{q}_{\sigma_i}, \boldsymbol{q}_{\sigma_{i+1}}) + \mathcal{L}(\boldsymbol{q}_{\sigma_n}, \boldsymbol{q}_{\sigma_1})$$

$$\text{subject to} \quad \boldsymbol{q}_i = (\boldsymbol{p}_i, \theta_i) \;\; i = 1, \ldots, n,$$



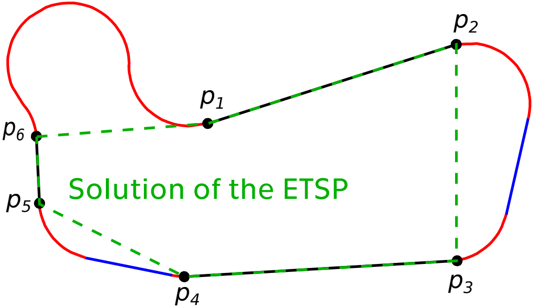
- Decoupled approaches – determining a sequence (e.g., using the Euclidean TSP) and headings.
  - Heuristic solution such as Alternating Algorithm (AA).
  - High-quality solution of the Dubins Touring Problem (DTP) using lower bounds.
- Transformation (sampling-based) approaches – transform the DTSP to the Generalized TSP.
- **Direct methods** – solve both optimization problems together:
  - Evolutionary (memetic) algorithms, **unsupervised learning**;
  - combinatorial metaheuristic with **continuous optimization of the underlying DTP**.

Alternating Algorithm (AA) – Decoupled Solution of the DTSP

`Alternating Algorithm` (AA) provides a solution of the DTSP for an even number of targets n.

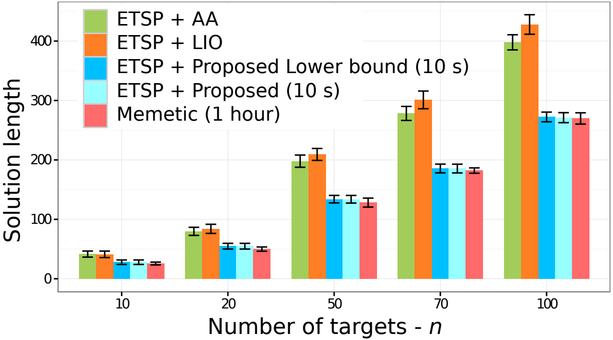
Savla, K., Frazzoli, E., Bullo, F.: *On the point-to-point and traveling salesperson problems for Dubins'vehicle*, **IEE American Control Conference**, 2005.

1. Solve the related Euclidean TSP.
   Relaxed motion constraints
2. Establish headings for even edges using straight line segments.
3. Determine optimal maneuvers for odd edges using the analytical form for Dubins maneuvers.
   Headings are known.



Example of Solution Quality of the DTSP

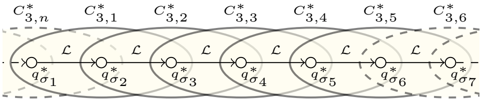
- Decoupled approaches use a single sequence found as a solution of the Euclidean TSP (ETSP):
  - AA – Alternating Algorithm (Savla et al., 2005);
  - LIO – Local Iterative Optimization (a variant of hill-climbing) (Váňa & Faigl, 2015); , Memetic – Zhang et al., 2014).
- Direct optimization of the sequence and headings
  - Memetic – Evolutionary method with continuous optimization (Zhang et al., 2014).

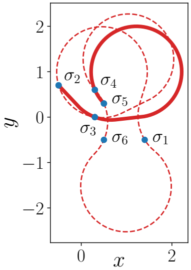


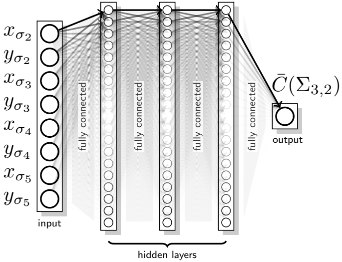
- Decoupled approach with high-quality solution of the **DTP** (based on lower bounds) provides competitive solutions to the direct method that also optimizes possible sequences.

Exploiting High-Quality Solution of the DTP for Solving the DTSP

- Motivation: Can we learn a surrogate model for quick assessment of sequence quality?
  - **Training data** based on high-quality solutions of the DTP enabled by the DIP (lower bounds).
  - Learn a surrogate model (e.g., multi-layer perceptron).
  - Generalization for arbitrary long sequences using sub-sequences of the defined size.





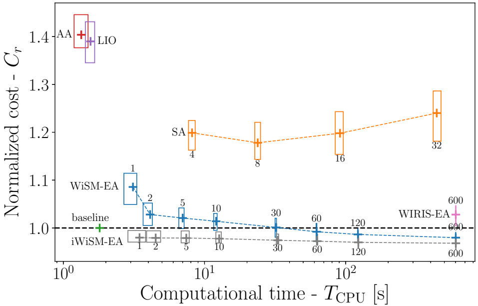


- A lower bound value of the optimal solution cost of the sequences Σ is estimated using estimates of the overlapping sub-sequences with the window of the size w: $\tilde{C}_w(\Sigma) = \frac{1}{w}\sum_{i=1}^{n}\overline{C}(\Sigma_{w,i})$.

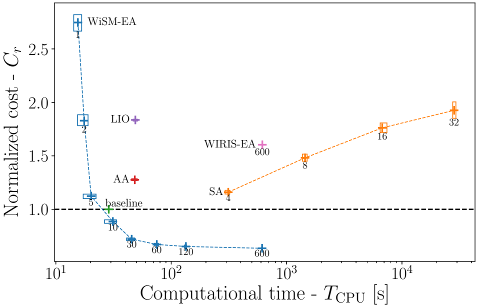
- Learned **Windowing Surrogate Model (WiSM)** can quickly assess any sequence of points as a solution cost of the corresponding Dubins Touring Problem.

WiSM: Windowing Surrogate Model for Solving the DTSP

- Use standard genetic operators with tournament selection and OX1 crossover method.
- The population is evaluated using learned surrogate model based on multi-layer perceptron.
- The surrogate model estimates solution cost of candidate sequences (instances of the DTP).
- Massive speedup of the evaluation yields improved solutions and scalability.



Instances with low density d and $n = 100$ target locations



Instances with high density d and $n = 500$ target locations

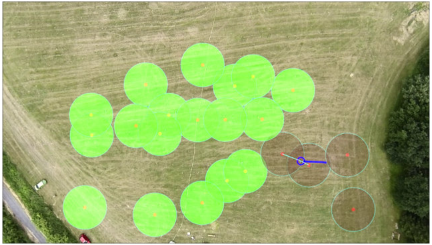
Drchal, J., Váňa, P., and Faigl, J.: *WiSM: Windowing Surrogate Model for Evaluation of Curvature-Constrained Tours with Dubins vehicle*, **IEEE Transactions on Cybernetics**, 2020.

Outline

- Motivation
- Dubins Vehicle Model and Routing
- Dubins Touring Problem (DTP)
- Dubins Traveling Salesman Problem (DTSP)
- Dubins Traveling Salesman Problem with Neighborhoods (DTSPN)
- Variable-speed TSP
- 3D Trajectories
- Unsupervised Learning


Dubins Traveling Salesman Problem with Neighborhoods

- In surveillance planning, we can save the travel cost by exploiting non-zero sensing range.
- The problem is to visit a set of regions $\boldsymbol{G} = \{R_1, \ldots, R_n\}$, where for each region R_i, we have to determine a particular point of the visit $p_i \in R_i$ and the DTSP becomes the **Dubins Traveling Salesman Problem with Neighborhoods** (**DTSPN**).



- DTSPN is an optimization problem over all permutations Σ, headings $\Theta = \{\theta_{\sigma_1}, \ldots, \theta_{\sigma_n}\}$ and points $P = (p_{\sigma_1}, \ldots, p_{\sigma_n})$ for the states $(q_{\sigma_1}, \ldots, q_{\sigma_n})$ such that $q_{\sigma_i} = (p_{\sigma_i}, \theta_{\sigma_i})$ and $p_{\sigma_i} \in R_{\sigma_i}$:

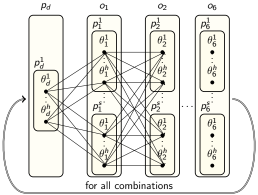
$$\begin{array}{ll} \text{minimize}_{\Sigma,\Theta,P} & \sum_{i=1}^{n-1} \mathcal{L}(q_{\sigma_i}, q_{\sigma_{i+1}}) + \mathcal{L}(q_{\sigma_n}, q_{\sigma_1}) \\ \text{subject to} & q_i = (p_i, \theta_i), p_i \in R_i \; i = 1, \ldots, n. \end{array}$$

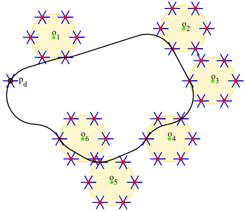
- $\mathcal{L}(q_{\sigma_i}, q_{\sigma_j})$ is the length of the Dubins maneuver connecting q_{σ_i} and q_{σ_j}.

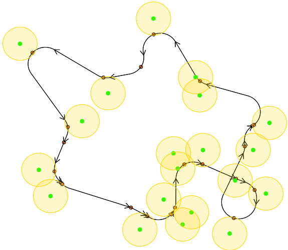
In addition to Σ and headings Θ, waypoint locations P have to be determined.

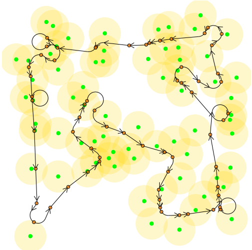
Existing Approaches to the DTSPN

- **Decoupled approaches** using the sequence of visits to the regions found as a solution of the Euclidean TSP(N);
- **Sampling-based transformation** to the GTSP.
  - Sampled locations, each sample with sampled possible headings.
- **Decoupled sampling-based** – for a given sequence, construct the search graph.









- **Soft-computing** techniques such as memetic algorithms.
- **Unsupervised learning** techniques.

Decoupled Approach to the DTSPN using Local Iterative Optimization

- For a given sequence, the solution of the DTSPN is continuous optimization of $2n$ **variables**, each from the interval $[0, 2\pi)$.
- p_i can be parametrized as a point on the bounday of the region R_i as a parameter $\alpha \in [0, 1)$ measuring a normalized distance on the boundary of R_i.
- Hill-climbing local optimization on heading value θ_i and waypoint p_i of visits to the region R_i.
- The multi-variable optimization is treated independenly for each particular variable θ_i and α_i.
- Performs "relatively good" but without any solution quality estimates.

Algorithm 2: Local Iterative Optimization (LIO) for the DTSPN

Data: Input sequence of the goal regions $\boldsymbol{G} = (R_{\sigma_1}, \ldots, R_{\sigma_n})$, for the permutation Σ

Result: Waypoints $(q_{\sigma_1}, \ldots, q_n)$, $q_i = (p_i, \theta_i)$, $p_i \in \delta R_i$

initialization() `// random assignment of` $q_i \in \delta R_i$;

while *global solution is improving* **do**

  for *every* $R_i \in \boldsymbol{G}$ **do**

    $\theta_i :=$ optimizeHeadingLocally(θ_i);

    $\alpha_i :=$ optimizePositionLocally(α_i);

    $q_i :=$ checkLocalMinima(α_i, θ_i);

  end

end

Váňa, P. and Faigl, J.: *On the Dubins Traveling Salesman Problem with Neighborhoods*, **IROS**, 2015, pp. 4029–4034.

- Can we assess the solution quality based on the lower bound estimates?

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Result: Waypoints $(q_{\sigma_1}, \ldots, q_n)$, $q_i = (p_i, \theta_i)$, $p_i \in \delta R_i$

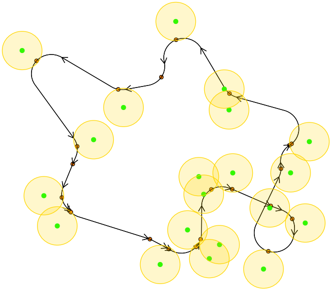
```
initialization() // random assignment of q_i ∈ δR_i;
while global solution is improving do
    for every R_i ∈ G do
        θ_i := optimizeHeadingLocally(θ_i);
        α_i := optimizePositionLocally(α_i);
        q_i := checkLocalMinima(α_i, θ_i);
    end
end
```

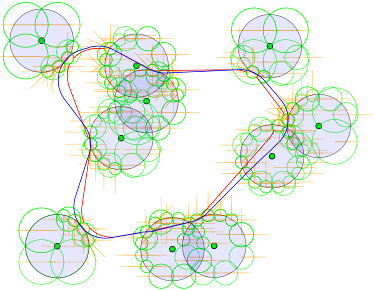
Váňa, P. and Faigl, J.: *On the Dubins Traveling Salesman Problem with Neighborhoods*, **IROS**, 2015, pp. 4029–4034.

- Can we assess the solution quality based on the lower bound estimates?

Lower Bound for the DTSP with Neighborhoods Generalized Dubins Interval Problem

- In the DTSPN, we need to determine the **headings** and also the **waypoint locations**.
- The **Dubins Interval Problem** (**DIP**) is not sufficient to provide tight lower-bound.





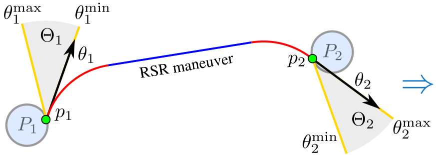
- **Generalized Dubins Interval Problem** (**GDIP**) can be utilized for the DTSPN similarly as the DIP for the DTSP.

Váňa and Faigl: *Optimal Solution of the Generalized Dubins Interval Problem*, **RSS** 2018, **best student paper finalist**.

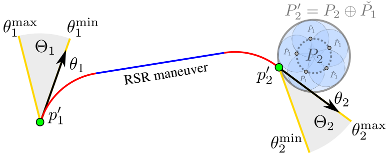
Generalized Dubins Interval Problem (GDIP)

- Determine the shortest Dubins maneuver connecting P_i and P_j given the angle intervals $\theta_i \in [\theta_i^{min}, \theta_i^{max}]$ and $\theta_j \in [\theta_j^{min}, \theta_j^{max}]$.

Full problem (GDIP)

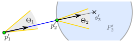


One-side version (OS-GDIP)

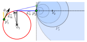


- **Optimal solution** – Closed-form expressions for (1–6) and convex optimization (7).

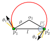
1) **S type**



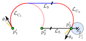
2) **CS type**



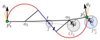
3) **C$_\psi$ type**



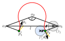
4) **CSC type**



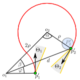
5) **CS$\overline{\text{C}}$ type**



6) **C$\overline{\text{C}}_\psi$C type**



7) **C$\overline{\text{C}}_\psi$ type**



Average computational time

Problem	Time [μs]	Ratio
Dubins maneuver	0.4	1.0
DIP	1.1	3.0
GDIP	5.4	14.5

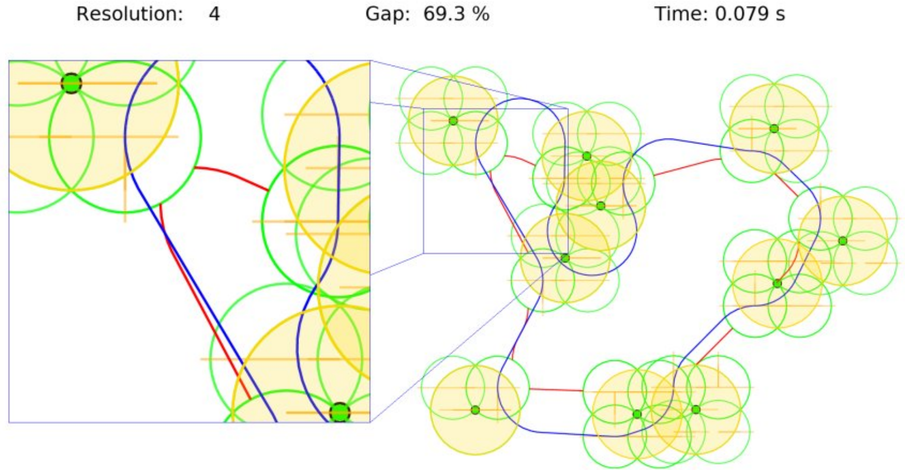
https://github.com/comrob/gdip

Váňa, P. and Faigl, J.: *Optimal Solution of the Generalized Dubins Interval Problem Finding the Shortest Curvature-constrained Path Through a Set of Regions*, **Autonomous Robots**, 44(7):1359-1376, 2020.

GDIP-based Informed Sampling for the DTSPN

- Iterative refinement of the neighborhood samples and heading samples.

Computational complexity of the solutions grows approximately as $\mathcal{O}(nk^{1.8})$.



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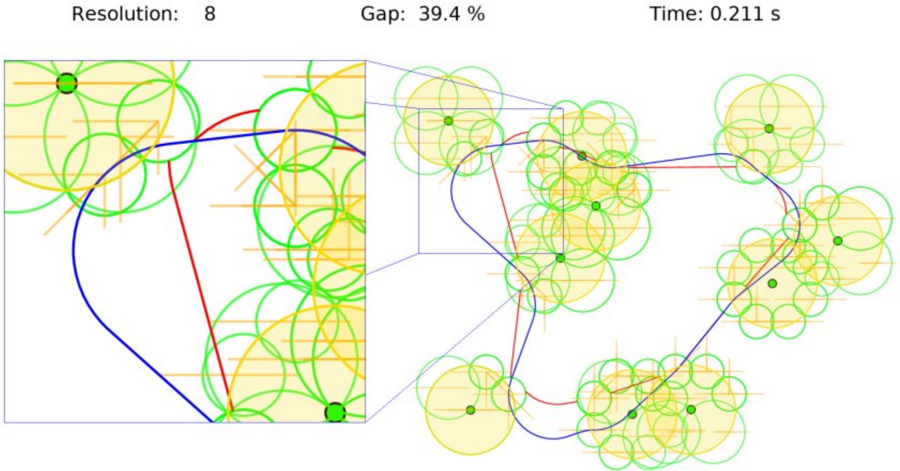

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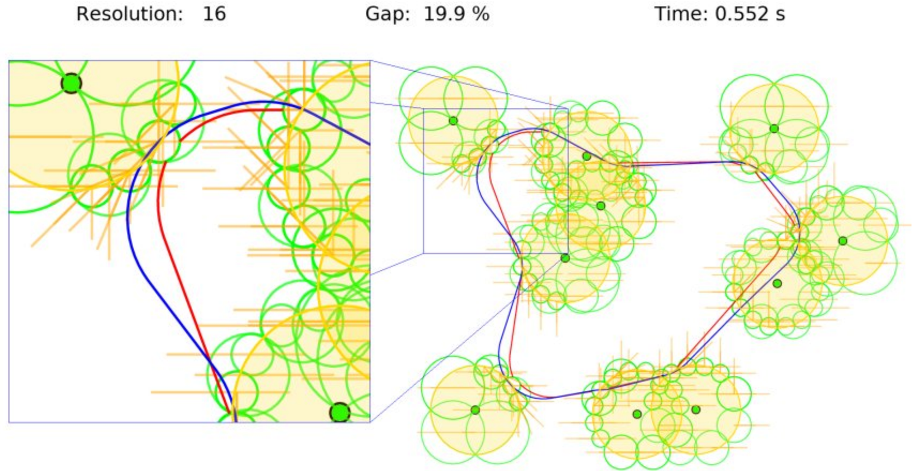

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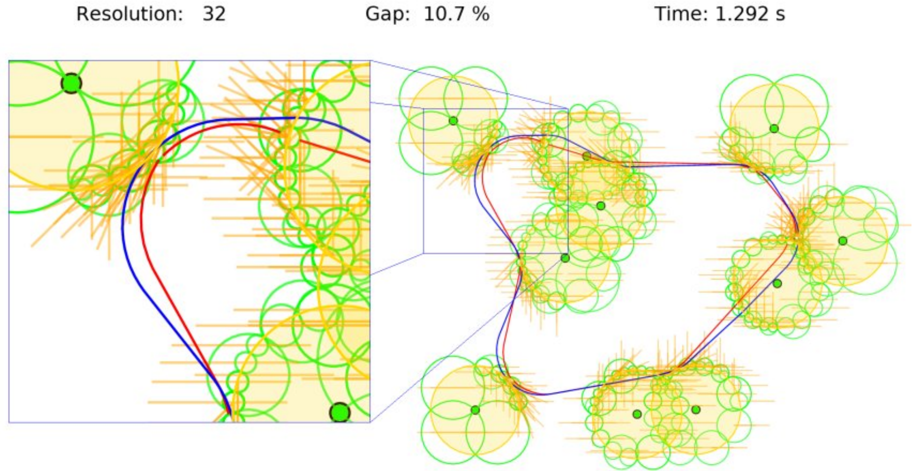

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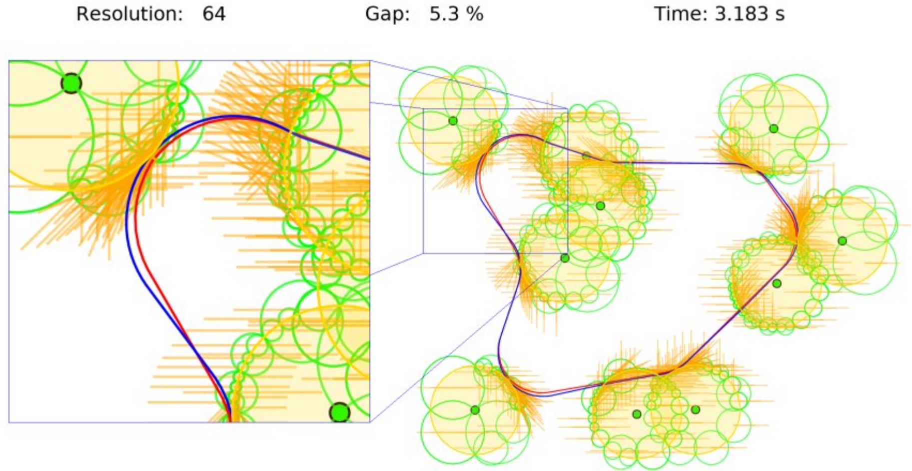

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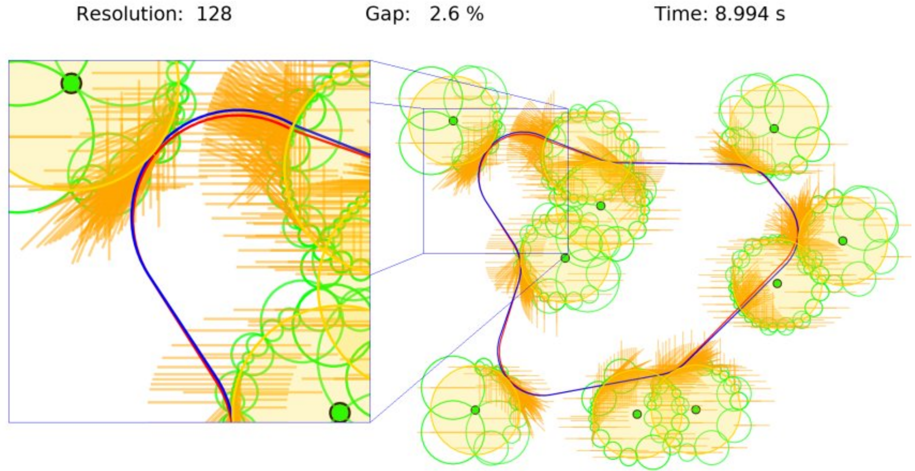

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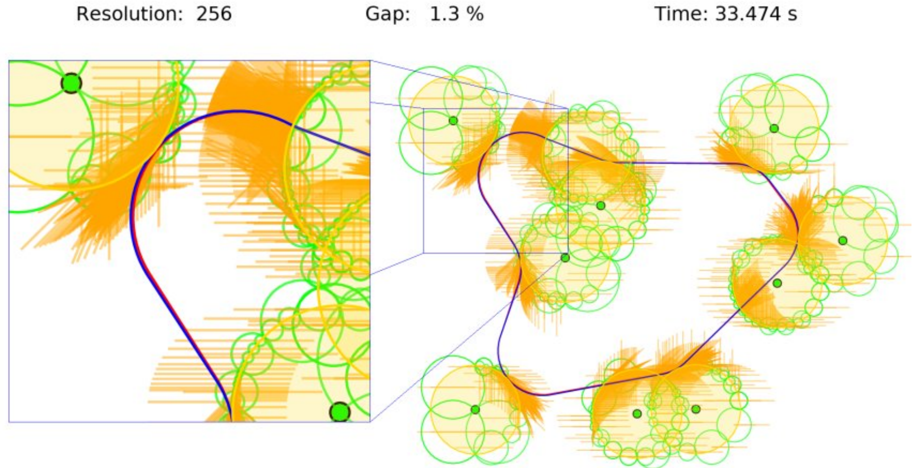
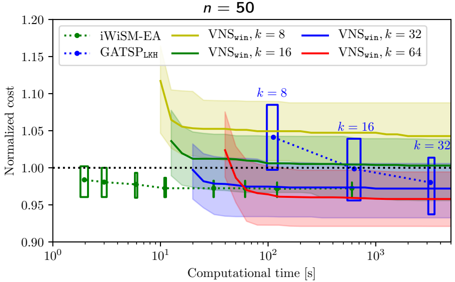
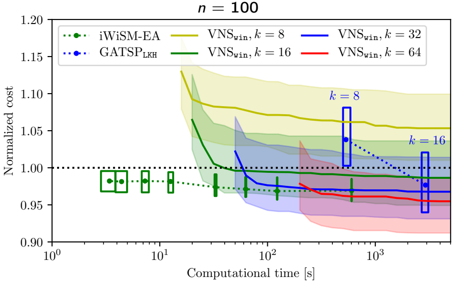


Optimal Solutions of the DTSP and DTSPN

- Lower bounds based on the DIP and GDIP provide lower bounds on the DTSP and DTSPN with a given sequence of visits to the target locations (regions).
- Lower bounds on the **Dubins Touring Problem** improve the solutions of the original problems, the space of possible sequences is not searched.
- Denser sampling in sampling-based methods would yield a better solution, but it is demanding.
- A quick assessment of the sequence quality can speed up finding the optimal sequence.





Faigl, J., Váňa, P., and Drchal, J.: *Fast Sequence Rejection for Multi-Goal Planning with Dubins Vehicle*, IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), 2020, pp. 6773–6780.

- Lower bounds can be used for bounding in the Branch-and-Bound to the DTSP(N).

Outline

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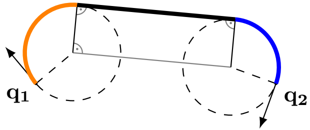
- 3D Trajectories
- Unsupervised Learning

Multi-radius Dubins Path

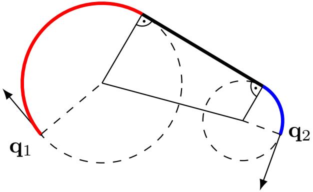
- **Dubins vehicle with different initial and final radii.**
- **Closed-form expression** exists.

  Computational requirements are competitive to the Dubins path
- **Time-optimal trajectory** is determined as optimization of the **turning radii (speed)**. *Larger radius → faster speed.*

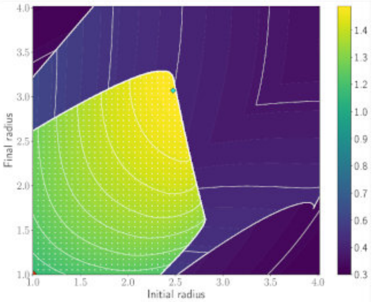
Dubins path (RSR type)

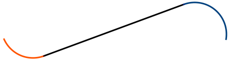


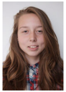
Multi-radius Dubins path (RSR type)



Speed up ratio compared to the regular Dubins path (yellow faster).



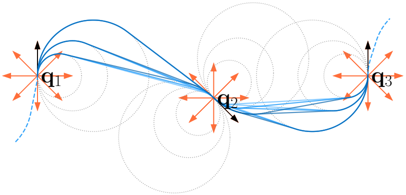


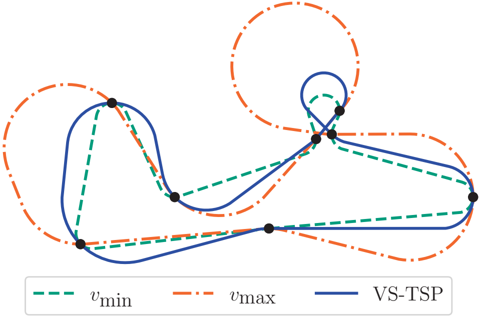


Kučerová, K., Váňa, P., and Faigl, J.: *On Finding Time-efficient Trajectories for Fixed-wing Aircraft Using Dubins Paths with Multiple Radii*, **35th Annual ACM Symposium on Applied Computing**, 2020, pp. 829–831.

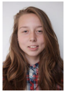
Multi-point Path and the Variable-Speed TSP (VS-TSP)

- Determine the fastest tour visiting a given sequence of locations by exploiting variable turning radii using **multi-radius Dubins path**.
- Accelerate on the straight segments.
- **Connection speed** discretized at the end-points to allow **feasible solution**.



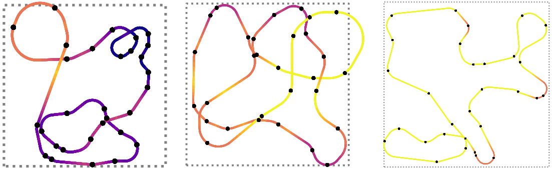


- Variable-Speed TSP with multi-radius path as evaluation of possible sequences.
- Improving an initial solution found by the **cheapest insertion**.
- Sequence search based on the **Variable Neighborhood Search (VNS)** with **fast sequence rejection** (IROS 2020).



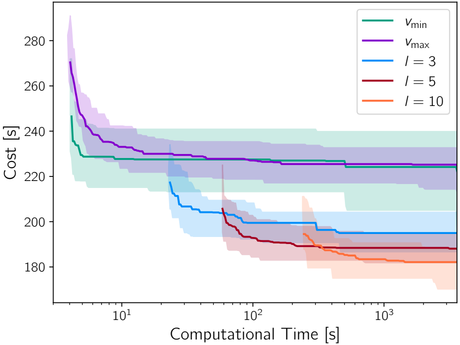
Influence of the Multi-radius Dubins path to the Variable-Speed TSP

- Sampling-based solution using the number of heading samples k and the number of speed samples l.
- **More samples** lead to **cost improvement** despite increasing computational difficulty.
- Depending on the density of locations, the vehicle speed ranges from **slowest to fastest**.

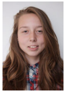


$d = 0.09$ $d = 0.05$ $d = 0.01$

Empirical Evaluation for 20 instances



$n = 30$ target locations with the density $d = 0.05$, $k = 10$ heading samples per each target location, and l speed samples.



Kučerová, K., Váňa, P., and Faigl, J.: *Variable-Speed Traveling Salesman Problem for Vehicles with Curvature Constrained Trajectories*, IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), 2021.

Outline

3D Dubins Path

- Determine the **shortest** 3D Dubins path for the vehicle with the state $q = (p, \theta, \psi)$, $p \in \mathbb{R}^3$, and heading θ and pitch ψ angles, $\theta, \psi \in \mathbb{S}^1$
- The position of the vehicle can be described as

$$\boldsymbol{r} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = v \begin{bmatrix} \cos\theta\cos\psi \\ \sin\theta\cos\psi \\ \sin\psi \end{bmatrix}.$$

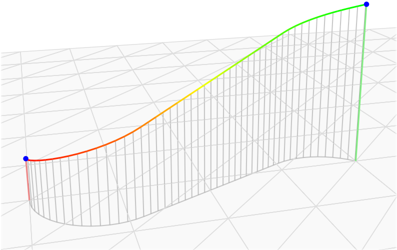
- The vehicle path $\boldsymbol{r} : [0, 1] \to \mathbb{R}^3$ is a curve from the class $\mathbb{C}^1$.
- **Motion constraints** are defined by the **limited curvature** $\kappa(t)$ given by the minimum turning radius ρ_{min} and **limited pitch angle** $\psi(t) \in [\psi_{\text{min}}, \psi_{\text{max}}]$.

$$\kappa(t) = \frac{|\dot{\boldsymbol{r}}(t) \times \ddot{\boldsymbol{r}}(t)|}{|\dot{\boldsymbol{r}}(t)|^3}, \quad \kappa(t) \le \rho_{\text{min}}^{-1}$$

- A closed-form solution is not known and the generation of 3D trajectory is based on heuristic and optimization methods using 2D Dubins maneuvers.

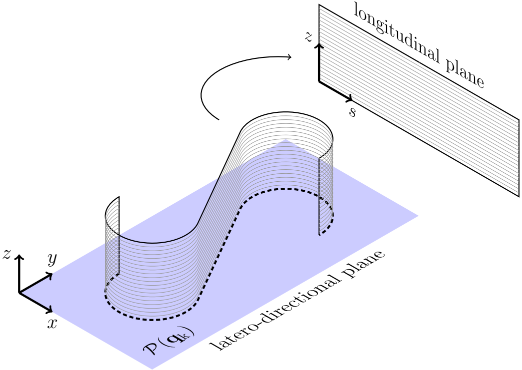
Existing Approaches to 3D Trajectory Planning

- **Geometrical approach** (Hota et al., 2010)
  Do not consider pitch angle constraint.
- **Dubins Airplane model** (Chitsaz et al., 2007)
  Allows abrupt changes of pitch angle.



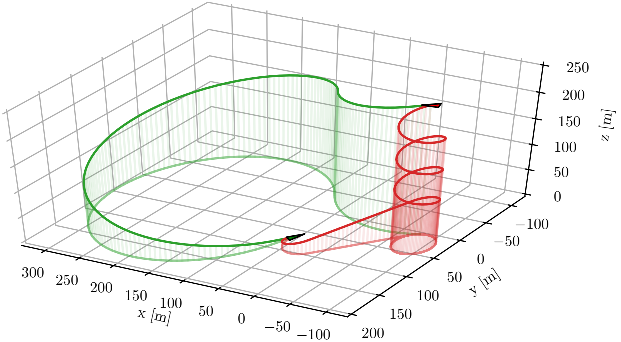
- **Real-Time Dynamic Dubins-Helix (RDDH)**
  Dubins-helix part is utilized for large altitude differences. (Wang et al., 2015)

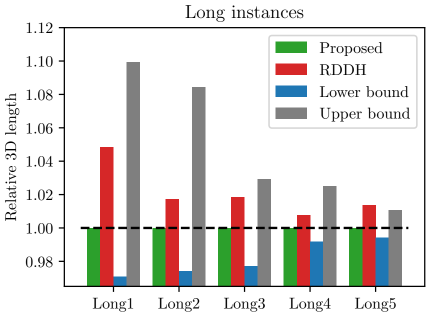
- **Decoupled approach using horizontal and vertical 2D Dubins paths** (Váňa et al., 2020)
  Relatively fast computation (around 0.2 ms).



Decoupled Approach for 3D Trajectory Planning

- Computing **horizontal** and **vertical** parts separately using 2D Dubins paths, optimizing turning radii to meet the **curvature constraint**, then combined into the final 3D Dubins path.
- **Lower** and **upper** bounds using vertical and horizontal radii. *Upper bound found in units of μs.*
- Less turns than the RDDH.

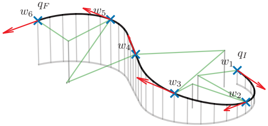


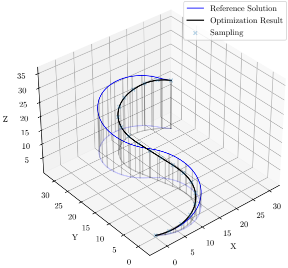


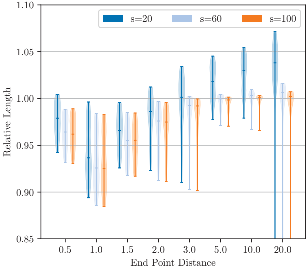
Váňa, P., Neto, A., Faigl, J., and Macharet, D.: *Minimal 3D Dubins Path with Bounded Curvature and Pitch Angle*, **IEEE International Conference on Robotics and Automation (ICRA)**, 2020, pp. 8497-8503.

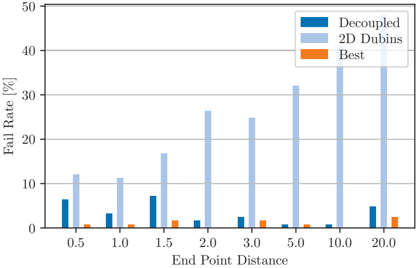
Improving Decoupled Approach using Non-linear Optimization

- Decoupled approach is used to initialize **non-linear optimization**.
- The encoding of the solution using direction vectors improved performance of the solver.









Herynek, J., Váňa, P., and Faigl, J.: *Finding 3D Dubins Paths with Pitch Angle Constraint Using Non-linear Optimization*, European Conference on Mobile Robots (ECMR), 2021.

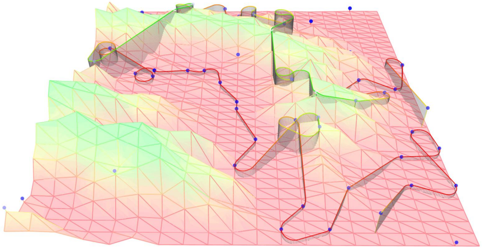
- Cost improvement is relatively small (about up to 5 %).
- Proposed combined initialization decrease fail rate, but still not usable for multi-goal trajectory.

3D Data Collection Planning with Dubins Airplane Model

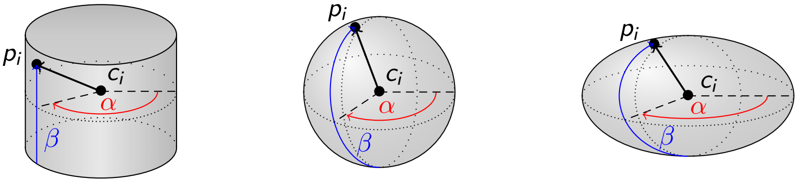
- **Dubins Airplane model** with the vehicle state $q = (p, \theta, \psi)$, $p \in \mathbb{R}^3$ and $\theta, \psi \in \mathbb{S}^1$, with the forward velocity v and control of the vehicle heading $|u_\theta| \leq 1$.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos\theta\cos\psi \\ \sin\theta\cos\psi \\ \sin\psi \\ u_\theta \rho^{-1} \end{bmatrix}.$$

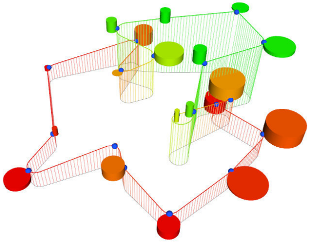
(Chitsaz et al., 2007)



- Pitch angle is not continuous (fast to compute).
- Waypoints can be parametrized as points on the 3D object boundary; we can employ LIO.



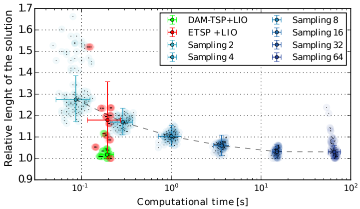
Solutions of the 3D-DTSPN

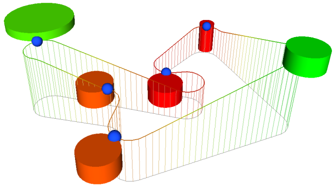


Algorithm 4: LIO-based Solver for 3D-DTSPN

Data: Regions $\mathcal{R}$
Result: Solution represented by $\mathcal{Q}$ and Σ
$\Sigma \leftarrow$ getInitialSequence($\mathcal{R}$);
$\mathcal{Q} \leftarrow$ getInitialSolution($\mathcal{R}, \Sigma$);
while *terminal condition* **do**
  $\mathcal{Q} \leftarrow$ optimizeHeadings($\mathcal{Q}, \mathcal{R}, \Sigma$);
  $\mathcal{Q} \leftarrow$ optimizeAlpha($\mathcal{Q}, \mathcal{R}, \Sigma$);
  $\mathcal{Q} \leftarrow$ optimizeBeta($\mathcal{Q}, \mathcal{R}, \Sigma$);
end
return $\mathcal{Q}, \Sigma$;

- Solutions based on LIO (ETSP+LIO), TSP with the travel cost according to Dubins Airplane Model (DAM-TSP+LIO), and sampling-based approach with transformation of the GTSP to the ATSP solved by LKH.





Váňa, P., Faigl, J., Sláma, J., and Pěnička, R.: *Data collection planning with Dubins airplane model and limited travel budget*
European Conference on Mobile Robots (ECMR), 2017.

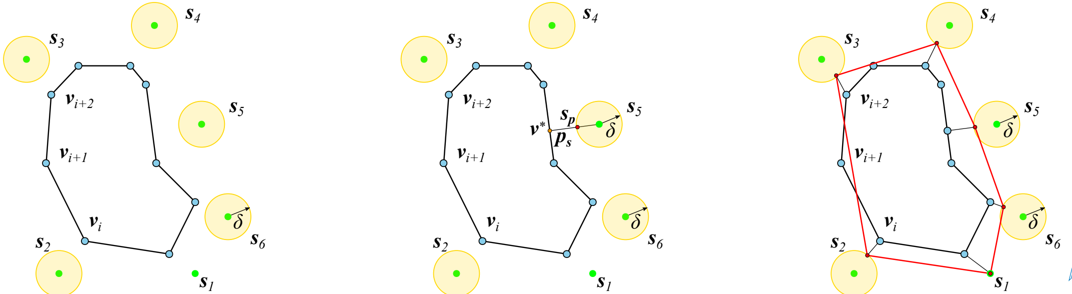
Outline

- Motivation
- Dubins Vehicle Model and Routing
- Dubins Touring Problem (DTP)
- Dubins Traveling Salesman Problem (DTSP)
- Dubins Traveling Salesman Problem with Neighborhoods (DTSPN)
- Variable-speed TSP
- 3D Trajectories
- Unsupervised Learning

Growing Self-Organizing Array (GSOA)

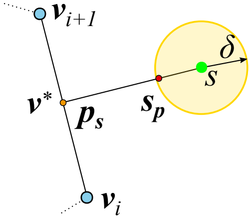
- **Growing Self-Organizing Array (GSOA)** is generalization of the Self-Organizing Maps to routing problems motivated by data collection planning, i.e., routing with neighborhoods.
- The GSOA is an array of nodes $\mathcal{N} = \{\nu_1, \ldots, \nu_M\}$ that evolves in the problem space using unsupervised learning. The array adapts to each target $s \in S$ (in a random order).
- For each target s a **new winner node $\boldsymbol{\nu}^*$** can be determined together with the corresponding $\boldsymbol{s}_p$, such that $\|(\boldsymbol{s}_p, \boldsymbol{s})\| \leq \delta(s)$.

  It adaptively adjusts the number of nodes.
- The winner and its neighborhoods are adapted (moved) towards $\boldsymbol{s}_p$.
- $\mathcal{N}$ encodes the sequence of visits Σ to the targets together with the corresponding waypoints.

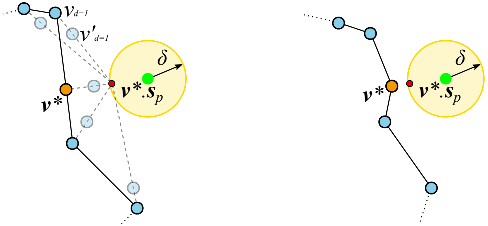


GSOA – Winner Selection and Its Adaptation

- Selecting winner node $\boldsymbol{\nu}^*$ for $\boldsymbol{s}$ and its waypoint $\boldsymbol{s}_p$



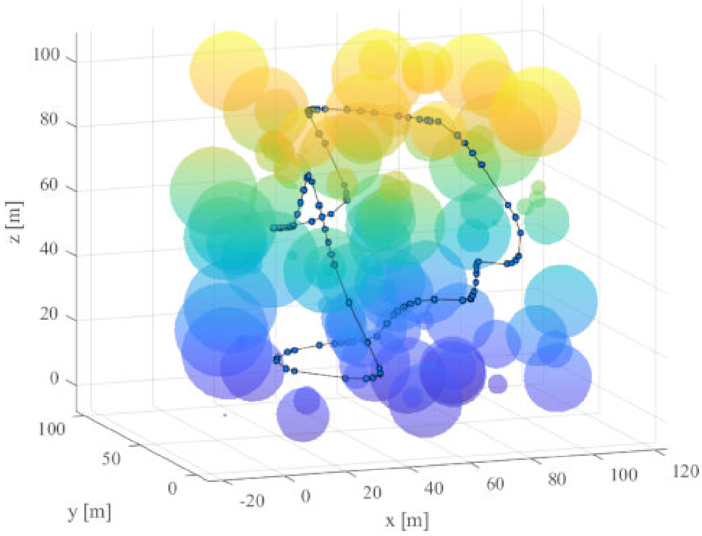
- Winner adaptation



- For each $s \in S$, we create new node ν^*, and therefore, all not winning nodes are removed after processing all locations in S (one learning epoch) to balance the number of nodes in the GSOA.
- After **each learning epoch**, the **GSOA encodes a feasible solution of the CETSP**.
- The power of adaptation is decreasing using a cooling schedule after each learning epoch.
- The GSOA converges to a stable solution in tens of epochs. Number of epochs can be set.

Faigl, J. (2018): GSOA: Growing Self-Organizing Array - Unsupervised learning for the Close-Enough Traveling Salesman Problem and other routing problems. Neurocomputing 312: 120-134 (2018).

GSOA Evolution in solving the 3D CETSP



GSOA in Solution of Routing Problems (with Neighborhoods)

- The flexiblity of the GSOA allows solving various routing problems, e.g., **Generalized TSP with Neighborhoods** and non-Euclidean variant such as the **TSPN on Sphere**.

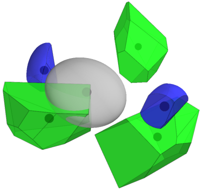
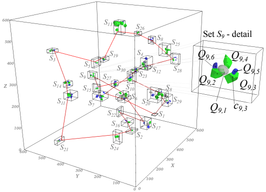
  Faigl, J., Deckerová, J., and Váňa, P.: *Fast Heuristics for the 3D Multi-Goal Path Planning based on the Generalized Traveling Salesman Problem with Neighborhoods*, **IEEE Robotics and Automation Letters**, 4(3):2439–2446, 2019.

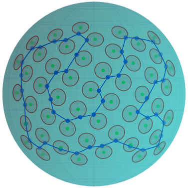
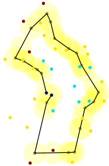
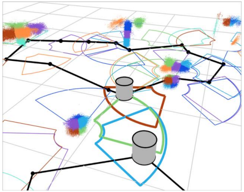
- Because $\mathcal{N}$ directly encodes the solution, the GDOA can address the **OP with Neighborhoods**.

  Faigl, J.: *On self-organizing maps for orienteering problems*, **International Joint Conference on Neural Networks (IJCNN)**, 2017, pp. 2611–2620.

- GSOA also allows solving problems with multiple vehicles.

  Best, G., Faigl, J., and Fitch, R.: *Online planning for multi-robot active perception with self-organising maps*, **Autonomous Robots**, 42(4):715–738, 2018.



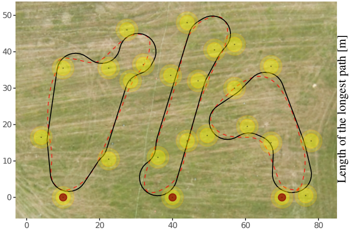
GSOA in Solution of Dubins Routing Problems

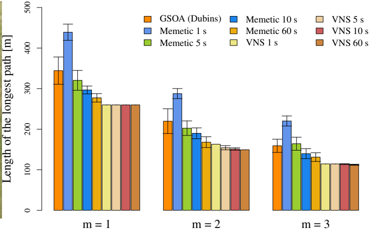
- A solution of the multi-vehicles **Dubins Traveling Salesman Problem with Neighborhoods** in less than 0.2 s, for m vehicles and sensing range δ.

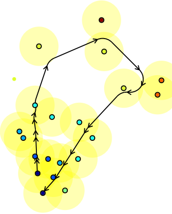
  Faigl, J., Váňa, P., Pěnička, R., and Saska, M.: *Unsupervised learning-based flexible framework for surveillance planning with aerial vehicles*, **Journal of Field Robotics**, 36(1):270–301, 2019.

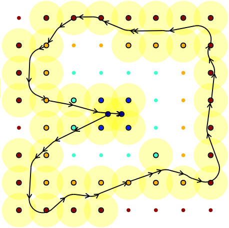
- Solution of the **Dubins Orienteering Problem with Neighborhoods** in tens of seconds that is about several magnitudes faster than using the VNS combinatorial metaheuristic.

  Faigl, J. and Pěnička, R.: *On close enough orienteering problem with Dubins vehicle*, **IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)**, 2017, pp. 5646–5652.





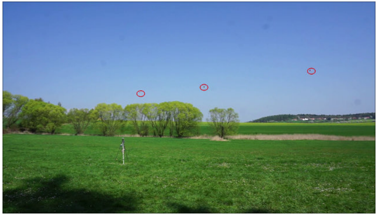


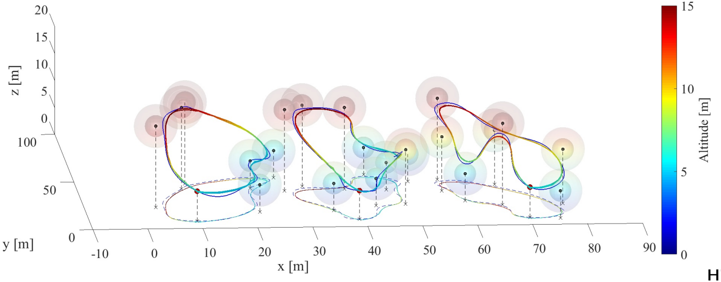


- Unsupervised learning can be considered as fast construction heuristic without formal solution quality guarantee, yet flexible enough to solve complex practical problems.

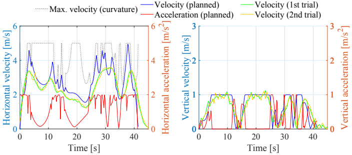
GSOA based Multi-goal Planning for Multi-rotor Aerial Vehicles

- Multi-rotor aerial vehicles can generally move in arbitrary direction.
  - They are not limited by the minimal turning radius ρ, they **can accelerate on straight segments and decelerate before turning.**
- Find a 3D smooth trajectory visiting a given set of 3D regions.
- Minimizes the **Travel Time Estimation** (TTE).
- Satisfies **limited velocity and acceleration** of the vehicle.





Velocity and acceleration profiles



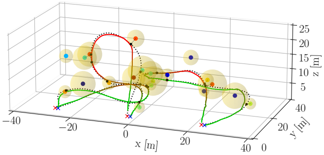


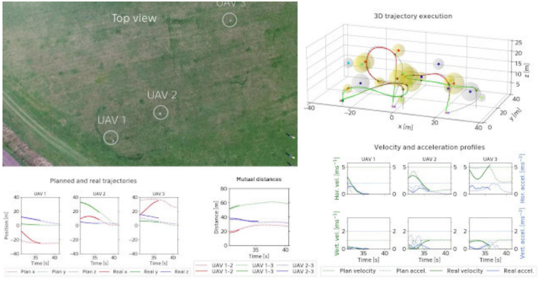
High altitudes changes saturate vertical velocity

Faigl, J. and Váňa, P.: *Surveillance Planning with Bézier Curves*. IEEE Robotics and Automation Letters, 3(2):750–757, 2018.

Multi-Vehicle Multi-Goal Planning with Limited Travel Budget – Curvature-Constrained Team Orienteering Problem (with Neighborhoods)

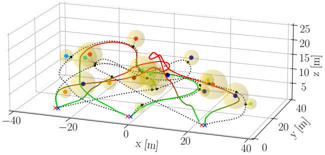
- Operational time of multi-rotor aerial vehicles is limited and only a subset of locations can be visited.
- Planning multi-goal trajectories as a sequence of Bézier curves.





Orienteering Problem with Bézier curves: Non-crossing field experiment with 3 multi-rotor drones

- Targets are mised in a case of colliding trajectories, because of local collision avoidance and optimal trajectory following.
- **There is a practical need to include coordination in multi-vehicle multi-goal trajectory planning**.



Faigl, J., Váňa, P., and Pěnička, R.: *Multi-Vehicle Close Enough Orienteering Problem with Bézier Curves for Multi-Rotor Aerial Vehicles*. **ICRA** 2019, pp. 3039–3044.

Summary

- Dubins vehicle model and Dubins routing problems
- **Dubins Touring Problem (DTP)**
- **Dubins Interval Problem (DIP)** (lower bound estimation to the DTP, DTSP)
- **Generalized Dubins Interval Problem (GDIP)** (lower bound estimation to the DTSPN)
- Dubins Traveling Salesman Problem (DTSP) and Dubins Traveling Salesman with Neighborhoods (DTSPN)
- Multi-radius Dubins path and Variable-Speed TSP (VS-TSP)
- 3D Dubins Path
- Unsupervised Learning in Dubins Routing Problems and 3D Surveillance Planning