Regrese a predikce pomocí fuzzy asociačních pravidel

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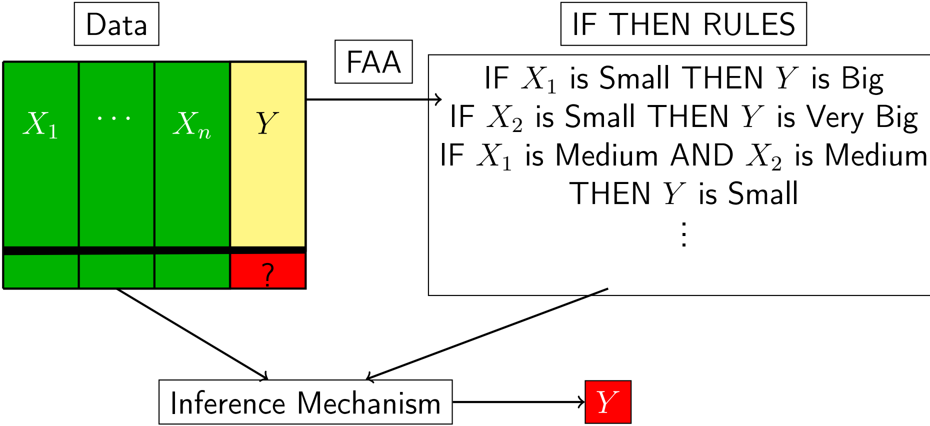
IRAFM

Regression based on rules



Short History Overview

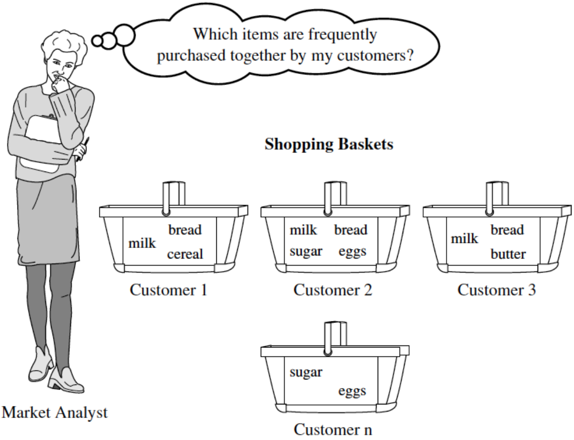
Fuzzy Associations

- 1966 Hájek et al - The GUHA method of automatic hypotheses determination
- 1978 Hájek and Havránek - Mechanizing Hypothesis Formation
- 1993 Agrawal et al - Mining association rules between sets of items in large databases.
- 2009 Ralbovský - Fuzzy Guha

Linguistic Summaries

- 1982 Yager - A new approach to the summarization of data
- 2000 Kacprzyk et al - A fuzzy logic based approach to linguistic summaries of databases

Association analysis



Association analysis example

Market Baskets

1. {beer, bread, soda}
2. {beer, bread, soda, cola, diapers}
3. {beer}
4. {beer, butter, soda}
5. {beer, butter, milk}
6. {beer, bread, soda, diapers}
7. {beer, bread, soda, diapers}
8. {bread, soda, diapers}

Support and Confidence

- $\text{supp}(\text{diapers} \rightharpoonup \text{beer}) = P(\{\text{diapers}, \text{beer}\}) = 3/8 = 0.375$
- $\text{conf}(\text{diapers} \rightharpoonup \text{beer}) = P(\{\text{diapers}, \text{beer}\} \mid \{\text{diapers}\}) = 3/4 = 0.75$
- $\text{conf}(\text{beer} \rightharpoonup \text{diapers}) = P(\{\text{diapers}, \text{beer}\} \mid \{\text{beer}\}) = 3/7 = 0.43$

Association Analysis - Four-fold table

	beer	diapers
o_1	1	0
o_2	1	1
o_3	1	0
o_4	1	0
o_5	1	0
o_6	1	1
o_7	1	1
o_8	0	1

	beer	¬ beer
diapers	$a = 3$	$b = 1$
¬ diapers	$c = 4$	$d = 0$

$$a = \sum \text{diapers} \wedge \text{beer}$$
$$b = \sum \text{diapers} \wedge \neg\text{beer}$$
$$c = \sum \neg\text{diapers} \wedge \text{beer}$$
$$d = \sum \neg\text{diapers} \wedge \neg\text{beer}$$

Support and Confidence

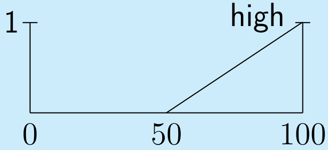
supp(diapers→beer)=P({diapers,beer})=a/(a+b+c+d)=0.375

conf(diapers→beer)=P({diapers,beer}|{diapers})=a/(a+b)=0.75

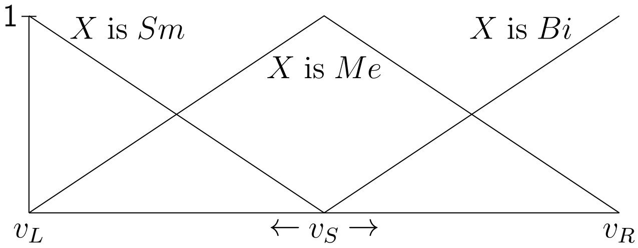
Fuzzy Attributes

	φ	ψ
o_1	0.1	0
o_2	0.9	1
o_3	1	0.5
o_4	0.5	0.3
o_5	0.7	0.4
o_6	0.8	0
o_7	1	0
o_8	0	0

$\varphi =$ "Temperature is high"



Semantics of Sm, Me, Bi



Linguistic hedges

Hedge	Abbreviation
extremely	Ex
significantly	Si
very	Ve

Table: Linguistic hedges with narrowing effect

Hedge	Abbreviation
rather	Ra
more or less	ML
roughly	Ro
quite roughly	QR
very roughly	VR

Table: Linguistic hedges with widening effect

Linguistic hedges

$$\underbrace{Ex \preceq Si \preceq Ve}_{\text{narrowing effect}} \preceq \langle\text{empty hedge}\rangle \preceq \underbrace{ML \preceq Ro \preceq QR \preceq VR}_{\text{widening effect}},$$

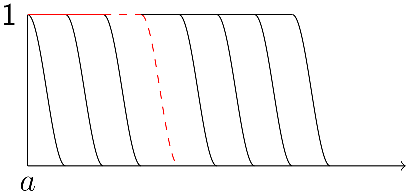
ν functions

$$\nu_{a,b,c}(x) = \begin{cases} 1, & c \leq x \\ 1 - \frac{(c-x)^2}{(c-b)(c-a)}, & b \leq x < c \\ \frac{(x-a)^2}{(b-a)(c-a)}, & a \leq x < b \\ 0, & x < a \end{cases}$$

where $a < b < c \in [0,1]$ and $\nu_{a,b,c} : [0,1] \to [0,1]$.

Fuzzy sets - all hedges

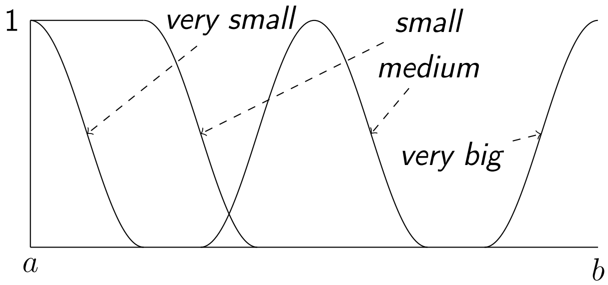
Figure: $\underbrace{Ex \preceq Si \preceq Ve}_{\text{narrowing effect}} \preceq \langle\text{empty hedge}\rangle \preceq \underbrace{ML \preceq Ro \preceq QR \preceq VR}_{\text{widening effect}},$



Fuzzy Sets

Definition

Fuzzy set A is a mapping from $\mathbb{R}$ to $[0,1]$.



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Fuzzy Attributes

	X
o_1	13.7
o_2	14.8
o_3	15.0
o_4	14.1
o_5	13.1
$\vdots$	$\vdots$
o_n	0.7

$\rightarrow$

ve sm X	sm X	me X	bi X	$\cdots$
0	0	0.2	0.8	
0	0	0	1	
0	0	0	1	
0	0	0.2	0.9	$\cdots$
0	0	0.1	0.7	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	
1	1	0	0	

Combining Fuzzy Attributes

T-norm

A *t-norm* is a function $\otimes : [0,1] \times [0,1] \mapsto [0,1]$ which satisfies the following properties:

- $a \otimes b = b \otimes a$
- $a \otimes b \leq a' \otimes b'$ if $a \leq a'$ and $b \leq b'$
- $(a \otimes (b \otimes c)) = ((a \otimes b) \otimes c)$
- $a \otimes 1 = a$

Negation

$\neg : [0,1] \mapsto [0,1]$, $\neg a = 1 - a$

T-conorm

- $a \oplus 0 = a$

Examples

Łukasiewicz

$$a \otimes_L b = \max\{0, a + b - 1\}$$

Product

$$a \otimes_P b = a \cdot b$$

Minimum

$$a \otimes_M b = \min\{a, b\} = a \wedge b$$

Residuated implication

Definition

A function $\Rightarrow: [0,1]^2 \mapsto [0,1]$ is called an *residuated* implication if there exists a t-norm $\otimes$ such that

$$x \Rightarrow y = \sup\{t \in [0,1] | x \otimes t \leq y\}.$$

Łukasiewicz

$a \Rightarrow_L b = 1 - a + b$

Product

$a \Rightarrow_P b = b/a$

Minimum

$a \Rightarrow_M b = b$

Fuzzy Four-fold table

For any two fuzzy attributes φ, ψ a generalized fuzzy four-fold table $E(\varphi, \psi, \otimes_a, \otimes_b, \otimes_c, \otimes_d, \neg)$ can be constructed, i.e.

$$E := \begin{array}{c|c|c} & \psi & \neg\psi \\ \hline \varphi & a & b \\ \hline \neg\varphi & c & d \end{array}, \tag{1}$$

where

$$\begin{aligned} a = & \sum_{o_i \in D} \varphi(o_i) \otimes_a \psi(o_i), \\ b = & \sum_{o_i \in D} \varphi(o_i) \otimes_b \neg\psi(o_i), \\ c = & \sum_{o_i \in D} \neg\varphi(o_i) \otimes_c \psi(o_i), \\ d = & \sum_{o_i \in D} \neg\varphi(o_i) \otimes_d \neg\psi(o_i). \end{aligned}$$

IRAFM

Mining of IF THEN rules

	A	B
o_1	0.1	0
o_2	0.9	1
o_3	1	0.5
o_4	0.5	0.3
o_5	0.7	0.4
o_6	0.8	0
o_7	1	0
o_8	0	0

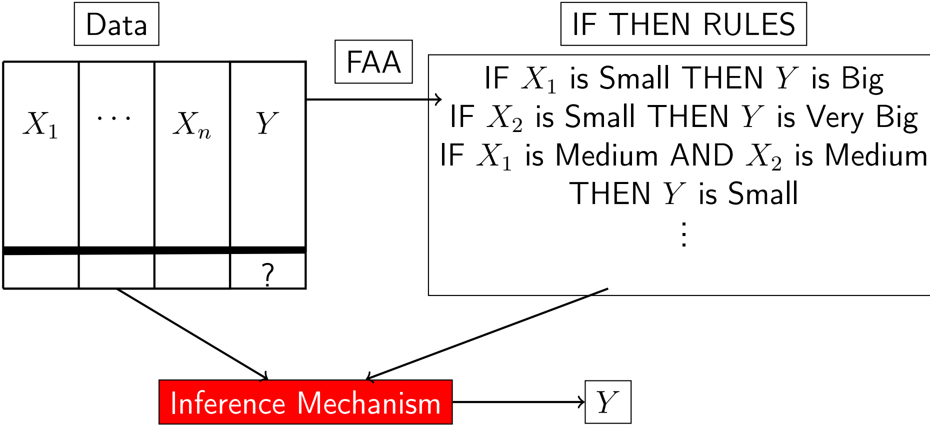
$$\mathrm{supp}(\varphi \rightharpoonup \psi) = \frac{\sum_{o_i} \varphi(o_i) \otimes \psi(o_i)}{n} \stackrel{?}{=} \frac{a}{a+b+c+d}$$

$$\mathrm{conf}(\varphi \rightharpoonup \psi) = \frac{\sum_{o_i} \varphi(o_i) \otimes \psi(o_i)}{\sum_{o_i} \varphi(o_i)} \stackrel{?}{=} \frac{a}{a+b}$$

Knowledge base - LD

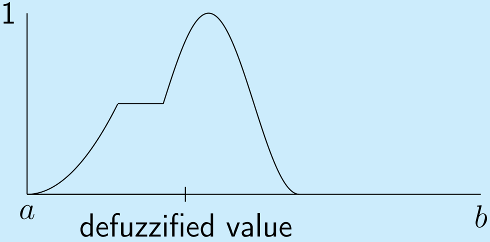
$\mathcal{R}_1, \ldots, \mathcal{R}_n$ such that $\mathrm{supp}(\mathcal{R}_i) > 0.02$, and $\mathrm{conf}(\mathcal{R}_i) > 0.9$

Final Inference

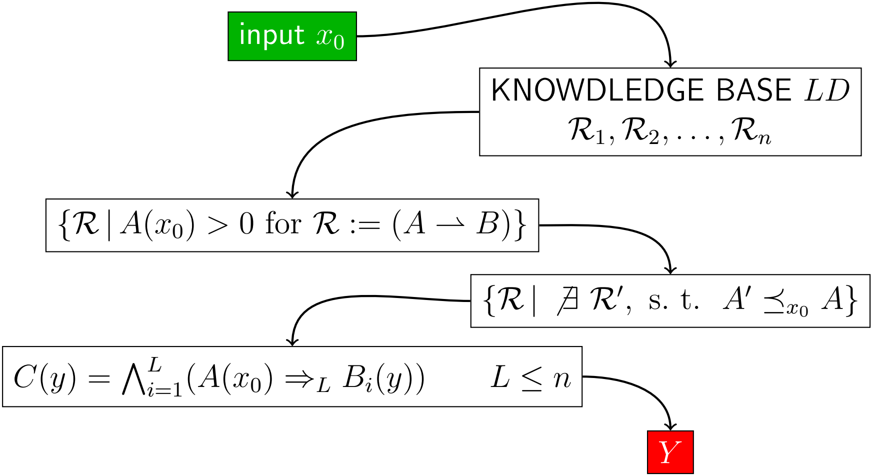


Defuzzification

Output fuzzy set



PbLD inference



Implicative Fuzzy Inference - Perception based Logic Deduction

Interpretation of the knowledge base

$$\hat{R}(x,y) = \bigwedge_{i=1}^{N} (A_i(x) \Rightarrow_L B_i(y))$$

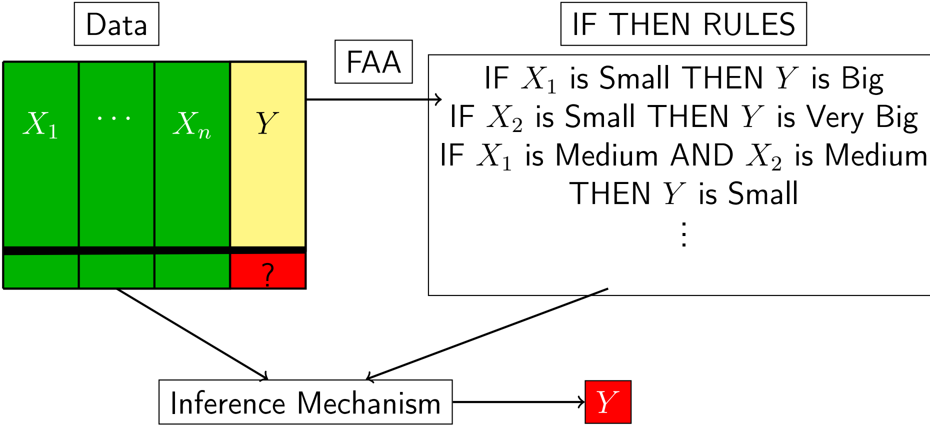
Ordering of antecedents

- partial ordering $\preceq$
- $L = \{A_i \mid \forall A_j \ (A_j \preceq_{x_0} A_i)$ then $(A_j = A_i)\}$

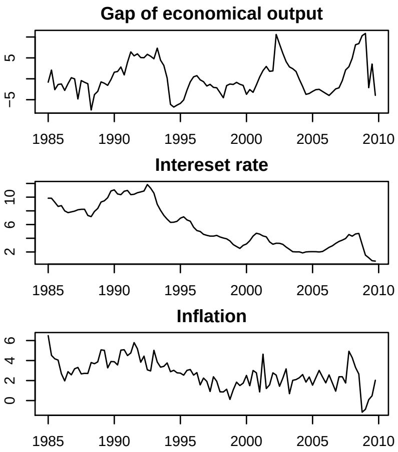
Output (with x_o as input)

$C(y) = \bigwedge_{i=1}^{L} (A(x_o) \Rightarrow_L B_i(y)) \qquad L \leq N$

Regression based on rules

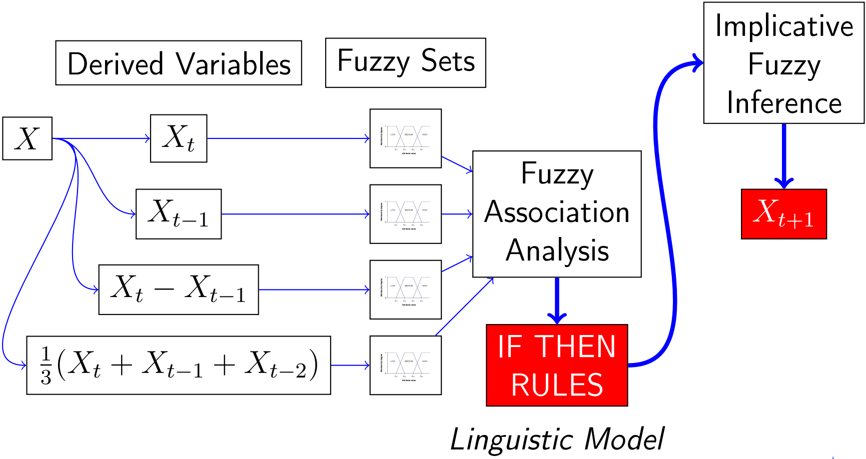


Time Series



Gap of economical output
5
−5
1985
1990
1995
2000
2005
2010
Intereset rate
10
6
2
1985
1990
1995
2000
2005
2010
Inflation
6
4
2
0
1985
1990
1995
2000
2005
2010

Model Construction



Derived Variables

	IR
o_1	13.7
o_2	14.8
o_3	15.0
o_4	14.1
o_5	13.1
$\vdots$	$\vdots$
o_n	0.7

$\Rightarrow$

IR_{diff}	$\text{IR}[i-1]$	$\text{IR}_{diff}[i-1]$	TIME
-	-	-	1
1.1	13.7	-	2
0.2	14.8	1.1	3
-0.9	15.0	0.2	4
-1.0	14.1	-0.9	5
$\vdots$	$\vdots$	$\vdots$	$\vdots$
0.2	0.9	0.5	n

Derived Variables

$$X_i^{S(h)}(t) := X_i(t-h)$$

$$X_i^{D(0,l,h)}(t) := X_i(t-h) - X_i(t-h-l)$$

$$X_i^{D(1,l,h)}(t) := X_i^{D(0,l,h)}(t-h) - X_i^{D(0,l,h)}(t-h-l)$$

$$X_i^{MA(n)}(t) := \frac{1}{n}\sum_{j=1}^{n} X_i(t-j)$$

$$X_{time}(t) := t$$

Membership Degrees to Fuzzy Sets

	IR
o_1	13.7
o_2	14.8
o_3	15.0
o_4	14.1
o_5	13.1
$\vdots$	$\vdots$
o_n	0.7

$\rightarrow$

ve sm IR	sm IR	me IR	bi IR
0	0	0.2	0.8
0	0	0	1
0	0	0	1
0	0	0.2	0.9
0	0	0.1	0.7
$\vdots$	$\vdots$	$\vdots$	$\vdots$
1	1	0	0

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IF THEN Rules

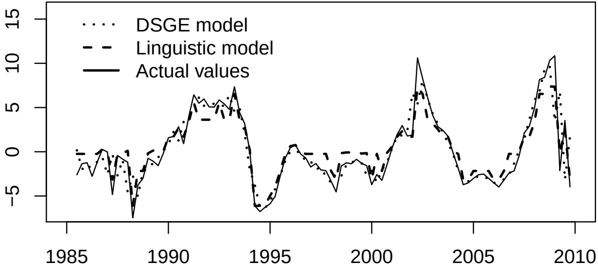
Example of mined rule

$$Rule1: \textbf{ IF } \mathrm{Time} \text{ is } Bi \textbf{ AND } \mathrm{G}^{S(10)} \text{ is } Bi \textbf{ THEN } \mathrm{G} \text{ is } Sm$$

$$Rule2: \textbf{ IF } \mathrm{IR}^{S(4)} \text{ is } Sm \textbf{ AND } \mathrm{G}^{S(10)} \text{ is } Bi \textbf{ THEN } \mathrm{G} \text{ is } Sm$$

Comparison with DSGE model

	Linguistic Model	DSGE Model
G	1.27	2.00
IR	0.64	0.50
I	0.86	0.93



Ensemble learning - models

$$
\begin{aligned}
Y &= f_1(X_1, \ldots, X_m) + E_1, \\
Y &= f_2(X_1, \ldots, X_m) + E_2, \\
&\vdots \\
Y &= f_r(X_1, \ldots, X_m) + E_r,
\end{aligned} \tag{2}
$$

$$
W_i = \frac{1}{|Y - f_i(X_1, \ldots, X_m)|},
$$

Ensemble learning - data

	X_1	X_2	...	X_m	Y	F_1	F_2	$\cdots$
o_1	e_{11}	e_{21}	...	e_{m1}	y_1	f_{11}	f_{21}	$\cdots$
o_2	e_{12}	e_{22}	...	e_{m2}	y_2	f_{12}	f_{22}	$\cdots$
$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
o_N	e_{1N}	e_{2N}	...	e_{mN}	y_N	f_{1N}	f_{2N}	$\cdots$

...

$\cdots$	F_r	W_1	W_2	$\cdots$	W_r
$\cdots$	f_{r1}	w_{11}	w_{21}	$\cdots$	w_{r1}
$\cdots$	f_{r2}	w_{12}	w_{22}	$\cdots$	w_{r2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\cdots$	f_{rN}	w_{1N}	w_{2N}	$\cdots$	w_{rN}

Table: A table representing an extended dataset D with regression models with their weights.

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Ensemble learning - rules

Rules

$$\textbf{IF } (X_i \text{ is } A_i \textbf{ AND } F_j \text{ is } A_j \ \ldots) \textbf{ THEN } W_i \text{ is } B, \qquad \text{for } A_i \in \mathcal{C}^{X_i} \quad F_j \in \mathcal{C}^{F_j},\ B \in \mathcal{C}^{W_i} \tag{3}$$

Ensemble Definition

$$\mathrm{ens_i} = \frac{\sum_j w_{ji} \cdot f_{ji}}{\sum_j w_{ji}}$$

$$\mathrm{ensMax}_i = f_{ji}, \quad \text{where } j = \arg\max_{j \in \{1,\ldots,r\}} \{w_{ji}\}$$

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Ensemble learning - Experiments

Example of an rule from ensemble

IF X_1 is Small AND MARS is Medium THEN W_1 is Significantly Small,

	LR	SVM	MARS	KNN	RT
average ranking	7.429	4.000	3.071	6.143	5.929
average RMSE	14.294	10.163	9.988	11.905	11.971

	Mean	ensMax	ens
average ranking	3.429	2.857	3.143
average RMSE	10.123	9.833	9.856

Table: Average ranking and root mean square errors for multiple regressions for datasets with average correlation between regression models *above* 0.5.

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Time Series Prediction Ensemble

"**IF** Strength of Seasonality is `Extremely Big` **AND** Kurtosis is `Quite Roughly Small` **THEN** Weight of the ARIMA method is `Big`."

Table: Number of Rules Generated by the Fuzzy GUHA Method and Number of Rules After Post-Processing.

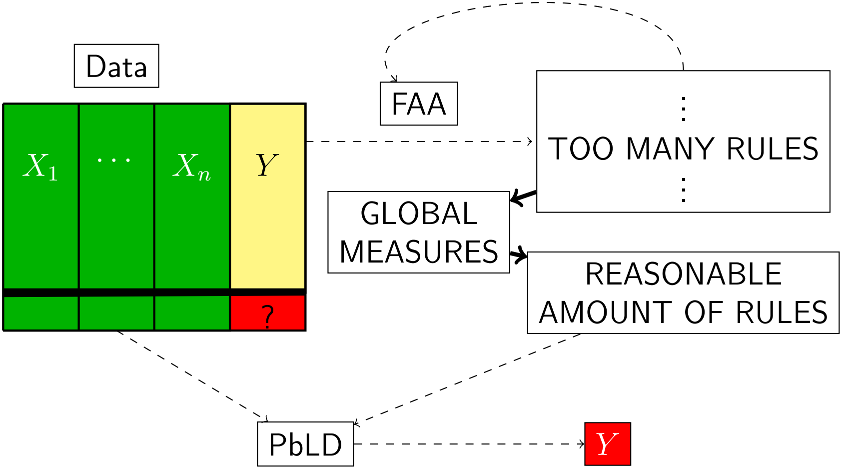
Method	Number of Rules After Application of Algorithms		
	Fuzzy GUHA	Redundancy Removal	Size Reduction
ARIMA	11206	9904	37
DT	63	29	10
ES	2244	1968	30
RW	153	49	14
RWd	2579	1941	45

Time Series Ensemble Results

Table: Average and Standard Deviation of the SMAPE Forecasting Errors.

Method	Error Average	Error Std.Dev.
ARIMA	14.58	16.77
DT	23.58	29.36
ES	14.31	16.44
RW	16.53	17.20
RWd	16.63	19.97
AM	14.73	16.88
FRBE	**13.93**	**15.47**

Too many rules



Coverage of Data

Definition

$$\mathrm{cov}_{\mathcal{O}}(LD) = \frac{1}{n}\sum_{j=1}^{n}\bigvee_{i=1}^{k} A_i(o_j)$$

Proposition

- $\mathrm{cov}_{\mathcal{O}}(LD) \in [0,1]$
- $\mathrm{cov}_{\mathcal{O}}(\emptyset) = 0$
- $LD' \subseteq LD$ then $\mathrm{cov}_{\mathcal{O}}(LD') \leq \mathrm{cov}_{\mathcal{O}}(LD)$
- If $S, T \in LD$ such that $S = A \rightharpoonup C$, $T = B \rightharpoonup C$ and $A \subset B$, then $\mathrm{cov}_{\mathcal{O}}(LD \setminus \{S\}) = \mathrm{cov}_{\mathcal{O}}(LD)$

Probabilistic coverage of data

Definition

$$\mathrm{cov}_{\mathcal{P}}(LD) = \int \cdots \int_S f_{\mathcal{O}}(X_1, \ldots, X_m)\, dX_1 \ldots dX_m.$$

Estimation based on data

$$\mathrm{cov}_{\mathcal{P}}(LD) = \frac{1}{n} \# \{o_j | A_i(o_j) > 0\}$$

Relationships

Proposition

- $\mathrm{cov}_{\mathcal{O}}(LD) \in [0, 1]$
- $\mathrm{cov}_{\mathcal{O}}(\emptyset) = 0$
- $LD' \subseteq LD$ then $\mathrm{cov}_{\mathcal{O}}(LD') \leq \mathrm{cov}_{\mathcal{O}}(LD)$
- If $S, T \in LD$ such that $S = A \rightharpoonup C$, $T = B \rightharpoonup C$ and $A \subset B$, then $\mathrm{cov}_{\mathcal{O}}(LD \setminus \{S\}) = \mathrm{cov}_{\mathcal{O}}(LD)$

Lemma

- $\mathrm{cov}_{\mathcal{O}}(LD) = 1$ implies $\mathrm{cov}_{\mathcal{P}}(LD) = 1$
- $\mathrm{cov}_{\mathcal{P}}(LD) = 1$ implies $\mathrm{cov}_{\mathcal{O}}(LD) > O$

Algorithm for probabilistic reduction

Informal description

- sample n rows from (training) data
- run n times the PbLD inference (only conditional firing)
- calculate the probability of a rule to be fired
- discard rule one by one till the desired threshold is reached
  - descending and ascending strategies

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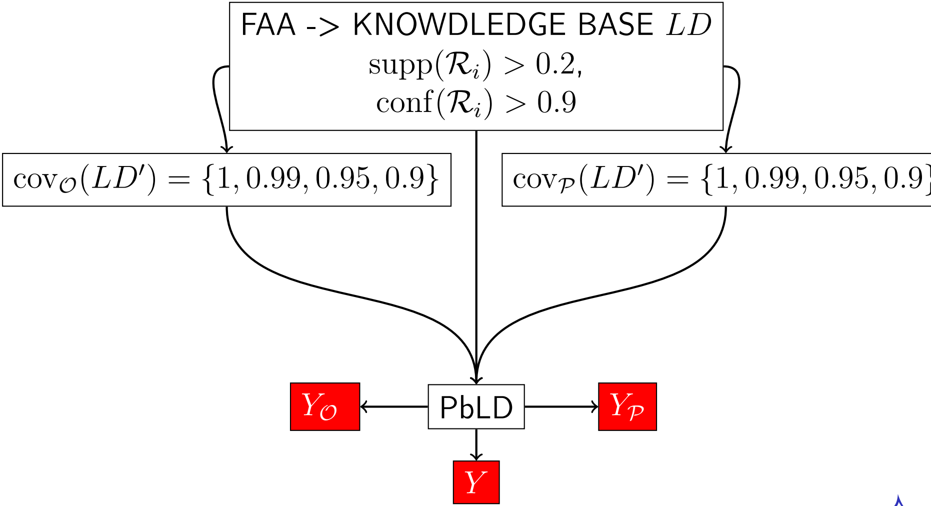
Experimental Comparison

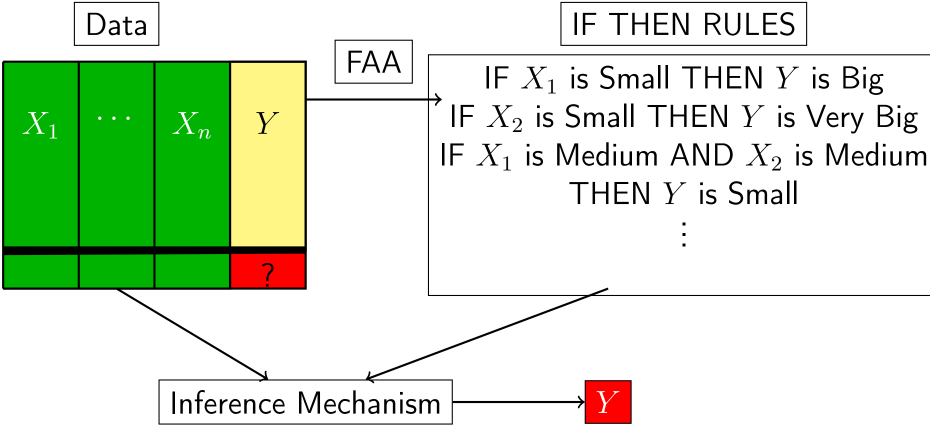


Experimental Results

Table: Results of reduction of rules based on $\text{cov}_{\mathcal{O}}$ and $\text{cov}_{\mathcal{P}}$

Dataset		FAA	$\text{cov}_{\mathcal{O}}$ best	$\text{cov}_{\mathcal{P}}$ best
auto-mpg	number	196	18	28
	error	4.78	4.83	4.78
automobile	number	2577	140	139
	error	8208.91	8430.8	5507.25
housing	number	2883	110	481
	error	8.06	8.02	8.06
airfoil	number	172	6	32
	error	8.02	8.02	8.02
yacht	number	110	19	8
	error	8.94	12.41	9.31

Regression based on rules



Fuzzy Association Analysis - experiments

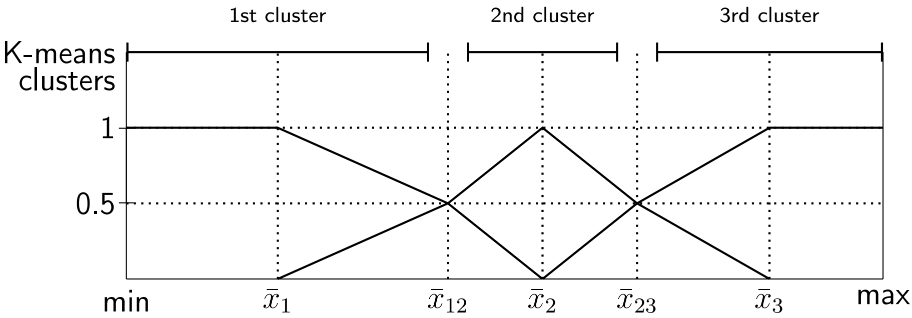


Figure: Triangular partition derived from k-means clustering. Where $\bar{x}_i$ is the mean of i-th cluster and $\bar{x}_{ij}$ is the mean of the i-th cluster maximum and the j-th cluster minimum.

Fuzzy Association Analysis - examples

- Rules $(A_1 \rightharpoonup B_1)$ and $(A_2 \rightharpoonup B_2)$
  - $\mathrm{conf}_l(A_1 \rightharpoonup B_1) < \mathrm{conf}_l(A_2 \rightharpoonup B_2)$
  - $\mathrm{conf}_m(A_1 \rightharpoonup B_1) > \mathrm{conf}_m(A_2 \rightharpoonup B_2)$
- Rules $(A_3 \rightharpoonup B_3)$ and $(A_4 \rightharpoonup B_4)$
  - $\mathrm{conf}_l(A_3 \rightharpoonup B_3) > \mathrm{conf}_l(A_4 \rightharpoonup B_4)$
  - $\mathrm{conf}_m(A_3 \rightharpoonup B_3) < \mathrm{conf}_m(A_4 \rightharpoonup B_4)$

	A_1	B_1	A_2	B_2	A_3	B_3	A_4	B_4
o_1	0.8	0.8	0.8	0.8	0.8	0.2	0.6	0.8
o_2	0.8	0.8	1	0.9	0.9	0.9	0.5	0.5

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Fuzzy Association Analysis - experiments

First n rules

The first n rules mined: $\mathcal{R}_c^n$, $\mathcal{R}_\otimes^n$ - $\otimes \in \{p, l, m\}$

Distances

$$K_n(\mathcal{R}_{\otimes_1}^n, \mathcal{R}_{\otimes_2}^n) = \frac{1}{n \cdot (n+1)} \sum_{r \in \mathcal{R}_{\otimes_1}^n \cup \mathcal{R}_{\otimes_2}^n} |rank_1(r) - rank_2(r)|,$$

$$D_n(\mathcal{R}_{\otimes_1}^n, \mathcal{R}_{\otimes_2}^n) = \#\{\mathcal{R}_{\otimes_1}^n \backslash \mathcal{R}_{\otimes_2}^n\}$$

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Fuzzy Association Analysis - experimental results

Table: Distances of rules with 1 antecedent mined from dataset `Yeast` with equi-width partition.

$D_{200} \backslash K_{200}$	$\mathcal{R}_c^{200}$	$\mathcal{R}_l^{200}$	$\mathcal{R}_m^{200}$	$\mathcal{R}_p^{200}$
$\mathcal{R}_c^{200}$	x	0.16	0.19	0.18
$\mathcal{R}_l^{200}$	22	x	0.15	0.11
$\mathcal{R}_m^{200}$	29	24	x	0.06
$\mathcal{R}_p^{200}$	25	12	14	x

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Fuzzy Quantifier

Fuzzy Quantifier q

A *fuzzy quantifier* q is a map defined on the set of fuzzy four-fold tables, i.e.

$$q : (\mathbb{R}^+)^4 \to [0,1],\ (a,b,c,d) \mapsto [0,1]$$

From tables to classes

	χ	$\neg\chi$
ζ	a'	b'
$\neg\zeta$	c'	d'

,

	ψ	$\neg\psi$
φ	a	b
$\neg\varphi$	c	d

,

Classes of Fuzzy Quantifiers

q is *implicational*

$a' \geq a, b' \leq b$ implies $q(a', b') \geq q(a, b)$.

q is *double implicational*

$a' \geq a$, $b' \leq b$ and $c' \leq c$ implies $q(a', b', c') \geq q(a, b, c)$.

q is *equivalence*

$a' \geq a$, $b' \leq b$, $c' \leq c$ and $d' \geq d$ implies $q(a', b', c', d') \geq q(a, b, c, d)$.

q is *ratio-implicational*

$a'b \geq ab'$ implies $q(a', b') \geq q(a, b)$.

Classes of Fuzzy Quantifiers - examples

q is *implicational*

$q(a,b) = \frac{a}{a+b}$

q is *double implicational*

$q(a,b,c) = \frac{a}{a+b+c}$

q is *equivalence*

$q(a,b,c,d) = \frac{a+d}{a+b+c+d}$

q is *ratio-implicational*

$q(a,b) = \frac{a}{a+\theta \cdot b}$, $\theta > 0$

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From fuzzy implications to fuzzy implicational quantifiers

Definition

A *fuzzy implication* is a binary operation $I : [0,1]^2 \to [0,1]$ for which $I(0,0) = I(1,1) = 1$, $I(1,0) = 0$ and $x' \leq x$, $y' \geq y$ implies $I(x', y') \geq I(x, y)$.

Lemma

Let I be fuzzy implication and $\varphi_1, \varphi_2 : \bar{\mathbb{R}} \to [0,1]$ be functions such that, for $i = 1, 2$,

1. φ_i is nonincreasing,
2. $\varphi_i(0) = 1$, and
3. $\varphi_i(\infty) = 0$.

Then $q_I : \bar{\mathbb{R}}^2 \to [0,1]$ defined by $q_I(a,b) := I(\varphi_1(a), \varphi_2(b))$ is a fuzzy implicational quantifier.

Ratio-implicational quantifiers

Lemma

Let I be a fuzzy implication then $q_I(a,b) = I\left(\frac{b}{a+b}, \frac{a}{a+b}\right)$ is a ratio-implicational quantifier with $0 = q(0,b)$ for all $b > 0$ and $1 = q(a,0)$ for all $a > 0$.

Lemma

Let q be a ratio-implicational quantifier with $0 = q(0,b)$ for all $b > 0$ and $1 = q(a,0)$ for all $a > 0$. Then there exists a fuzzy implication I_q such that

$$q(a,b) = I_q\left(\frac{b}{a+b}, \frac{a}{a+b}\right).$$

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Quality measures from fuzzy four fold table

When?

$$\mathrm{supp}(\varphi \rightharpoonup \psi) = \frac{\sum_{o_i} \varphi(o_i) \otimes \psi(o_i)}{n} \stackrel{?}{=} \frac{a}{a+b+c+d}$$

$$\mathrm{conf}(\varphi \rightharpoonup \psi) = \frac{\sum_{o_i} \varphi(o_i) \otimes \psi(o_i)}{\sum_{o_i} \varphi(o_i)} \stackrel{?}{=} \frac{a}{a+b}$$

	ψ	$\neg\psi$
φ	a	b
$\neg\varphi$	c	d

$$\begin{aligned} \mathsf{a} &= \sum_{o_i \in D} \varphi(o_i) \otimes_a \psi(o_i) \\ \mathsf{b} &= \sum_{o_i \in D} \varphi(o_i) \otimes_b \neg\psi(o_i), \\ \mathsf{c} &= \sum_{o_i \in D} \neg\varphi(o_i) \otimes_c \psi(o_i), \\ \mathsf{d} &= \sum_{o_i \in D} \neg\varphi(o_i) \otimes_d \neg\psi(o_i). \end{aligned}$$

Involutively dual t-norms

Definition

Let $\otimes_1$ and $\otimes_2$ are two t-norms. Then we say they are *involutively dual* if the following holds:

$$(x \otimes_1 y) + x \otimes_2 (1 - y) = x, \qquad \text{for } x, y \in [0, 1].$$

Example

$$T_p^F(x, y) = \log_p\left(1 + \frac{(p^x - 1)(p^y - 1)}{p - 1}\right), \qquad \text{where } p \in [0, \infty].$$

$$T_p^F(x, y) + T_{1/p}^F(x, 1 - y) = x.$$

Equality for involutively dual t-norms

Confidence

If $\otimes$ is $\otimes_a$ and $\otimes_b$ is involutively dual t-norm then:

$$\mathrm{conf}(\varphi \rightharpoonup \psi) = \frac{\sum_{o_i} \varphi(o_i) \otimes \psi(o_i)}{\sum_{o_i} \varphi(o_i)} = \frac{a}{a+b}$$

Support

$$\mathrm{supp}(\varphi \rightharpoonup \psi) = \frac{\sum_{o_i} \varphi(o_i) \otimes \psi(o_i)}{n} \stackrel{?}{=} \frac{a}{a+b+c+d}$$

Only Product

If $\otimes_a = \otimes_b$, and $a+b+c+d = n$ then $\otimes_a = \otimes_b = \otimes_c = \otimes_d = \odot$

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Rule base measures based on single rule validity

Current work

$$\mathrm{Inacc} = 1 - \frac{|\mathcal{O}^+|_\Sigma - |\mathcal{O}^-|_\Sigma}{|\mathcal{O}|}, \quad \mathrm{Impr}_1 = 1 - \frac{|\mathcal{O}^+|_\Sigma}{|\mathcal{O}|}, \qquad (4)$$

where $\mathcal{O}^+$ and $\mathcal{O}^-$ are fuzzy subsets of $\mathcal{O}$ such that:

- $\mathcal{O}^+(o)$ is a degree in which a *rule base R is valid for o*,
- $\mathcal{O}^-(o)$ is a degree in which a *rule base R is not valid for o*,

Conclusion and Future work

Future work for you

- lfl: Linguistic Fuzzy Logic (R package on CRAN)
  > install.packages("lfl")
  > frbe(d,h=10)

Thank you for your attention!

- Questions now or pavel.rusnok@osu.cz

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