

# Transductive active learning for fine-tuning large (language) models

Jonas Hübötter

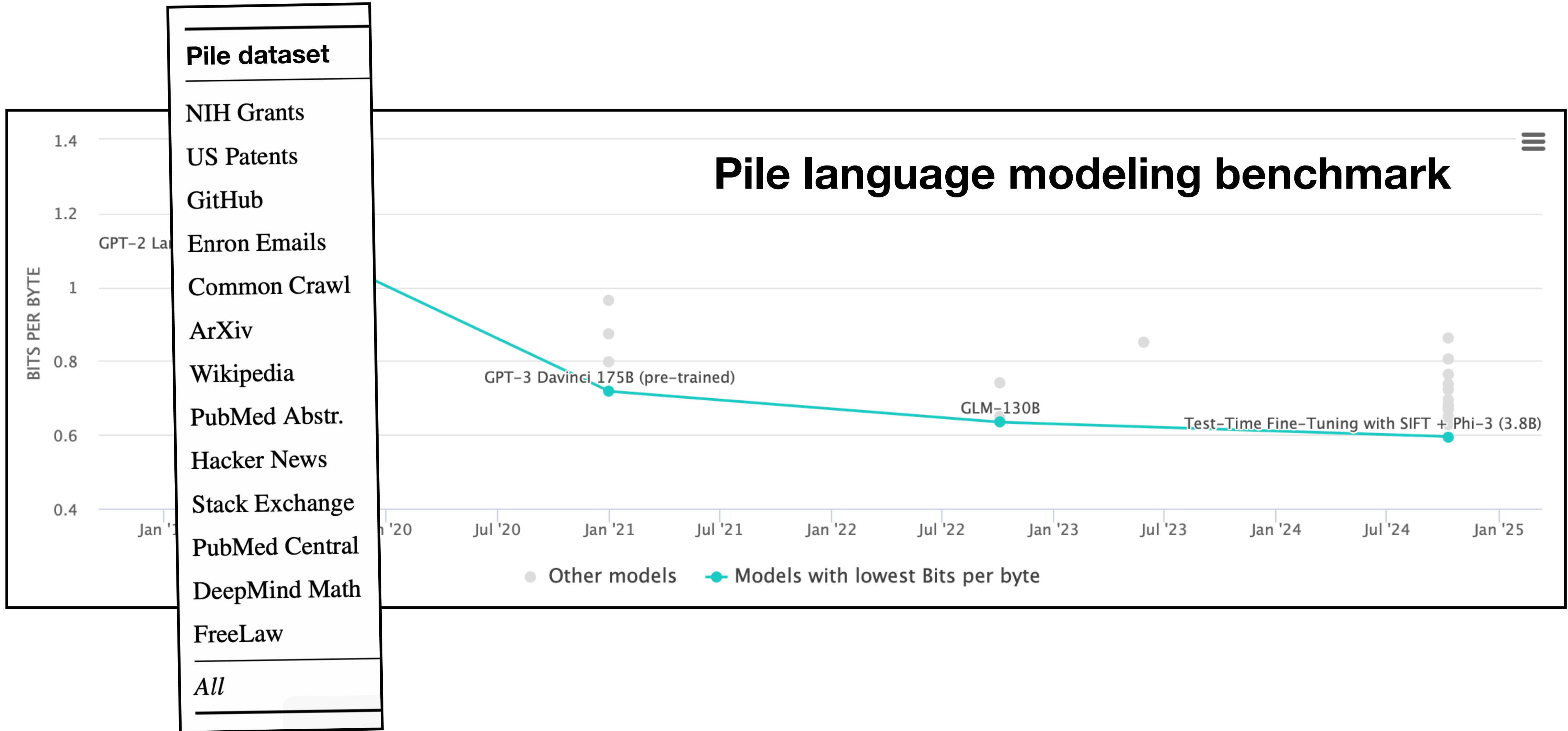


# Efficiently learning at test-time with LLMs

~~Transductive active learning for fine-tuning  
large (language) models~~

Jonas Hübötter





[H, Bongni, Hakimi, Krause; preprint]

Train

Test

# Local learning (at test-time)

known!

Training data  
 $\mathcal{D} = \{(\mathbf{x}_i, y_i)\}_{i=1}^n$

Learnt model  
 $f : \mathcal{X} \rightarrow \mathcal{Y}$

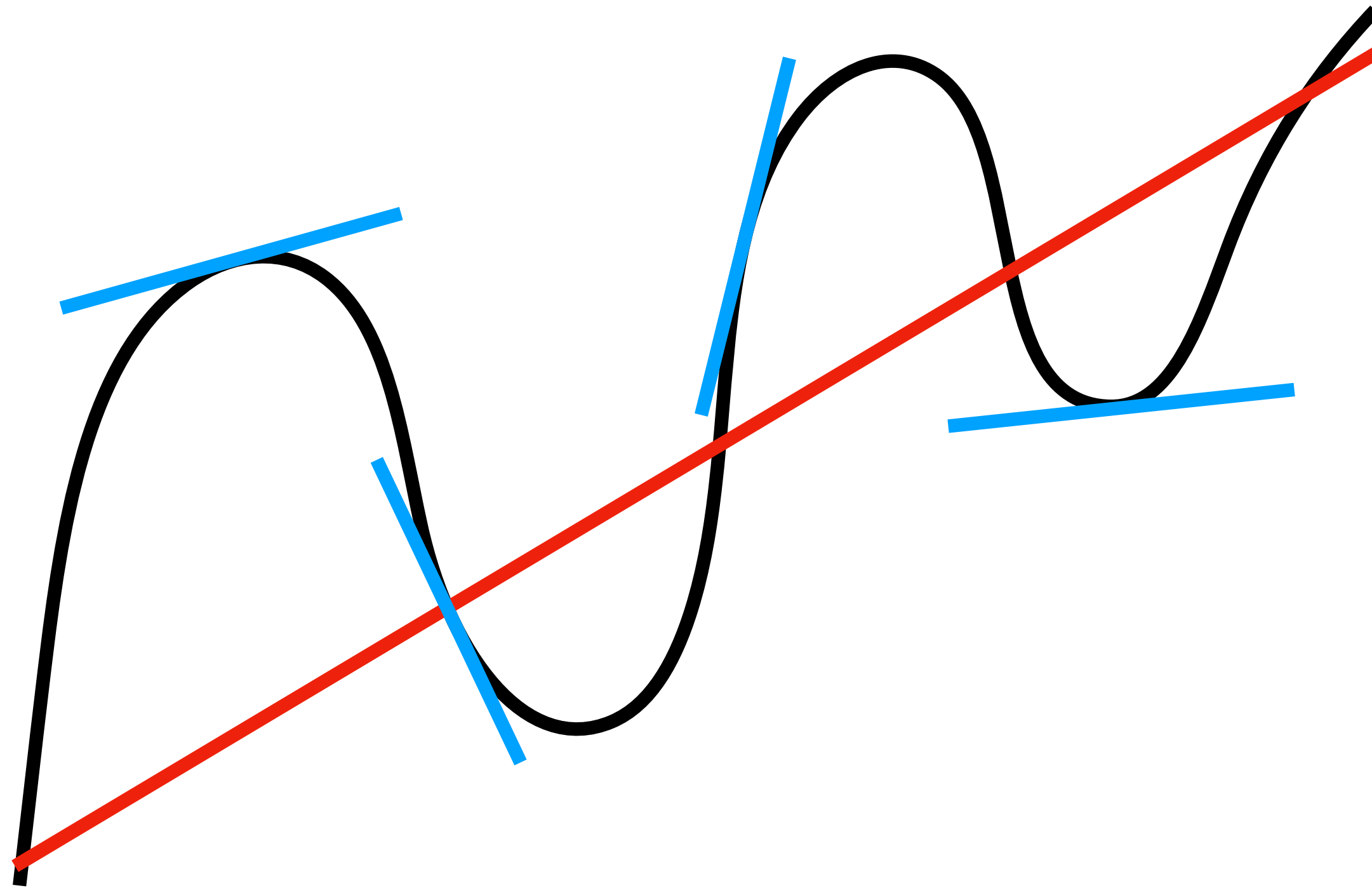
Test instance  $\mathbf{x}^*$

Prediction  
 $f(\mathbf{x}^*)$

# A story of curve fitting



# A story of curve fitting



## Remedies:

- Parametric models
  - polynomial regression
  - neural networks
- Non-parametric models
  - kernel (ridge) regression
  - k-nearest neighbor
- **Local** models
  - local linear regression
  - ...

# A story of curve fitting

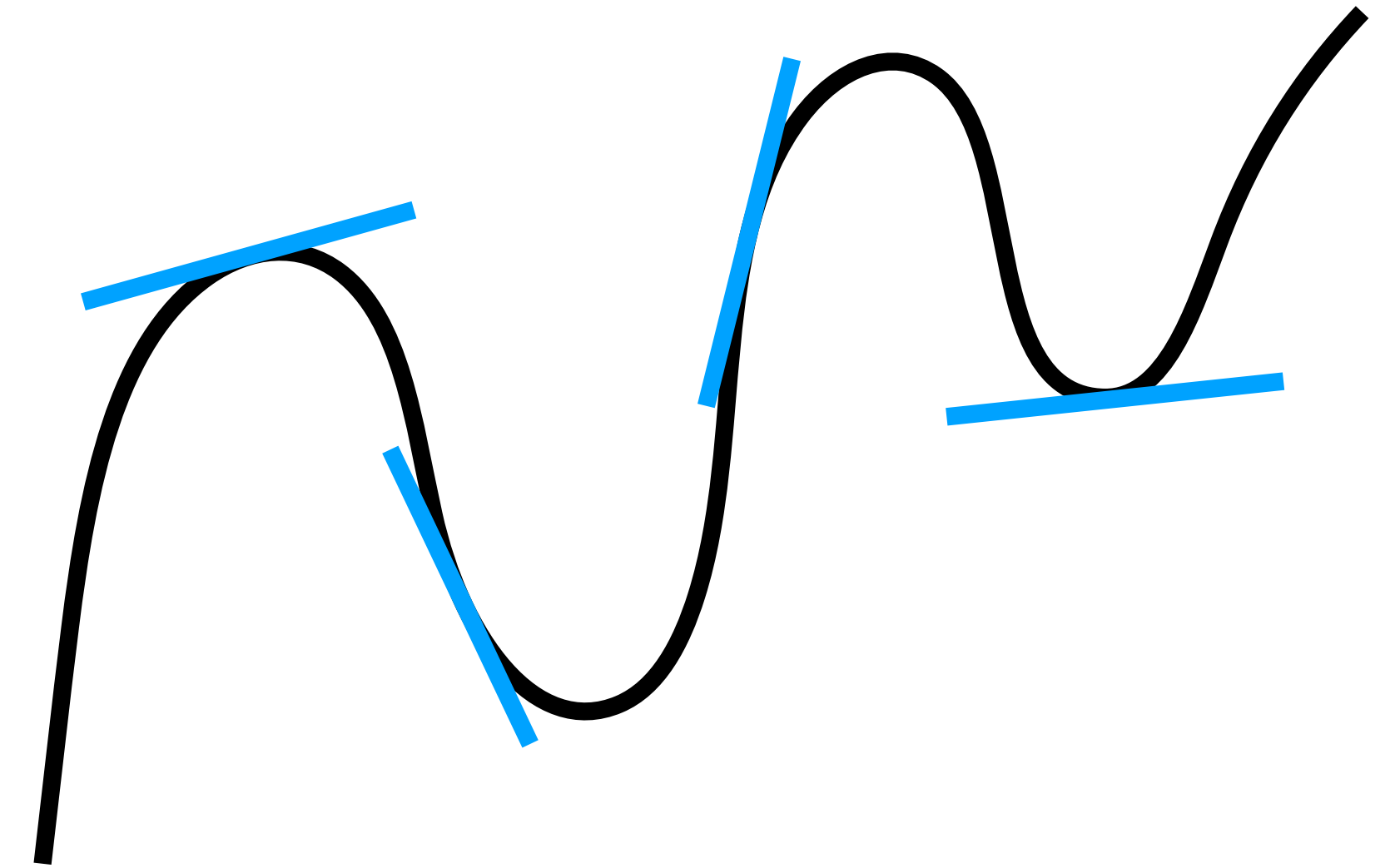
Local models have two components:

- *Parametric* “controller”  
linear regression  
...
- *Non-parametric* “memory”  
k-nearest neighbor  
...

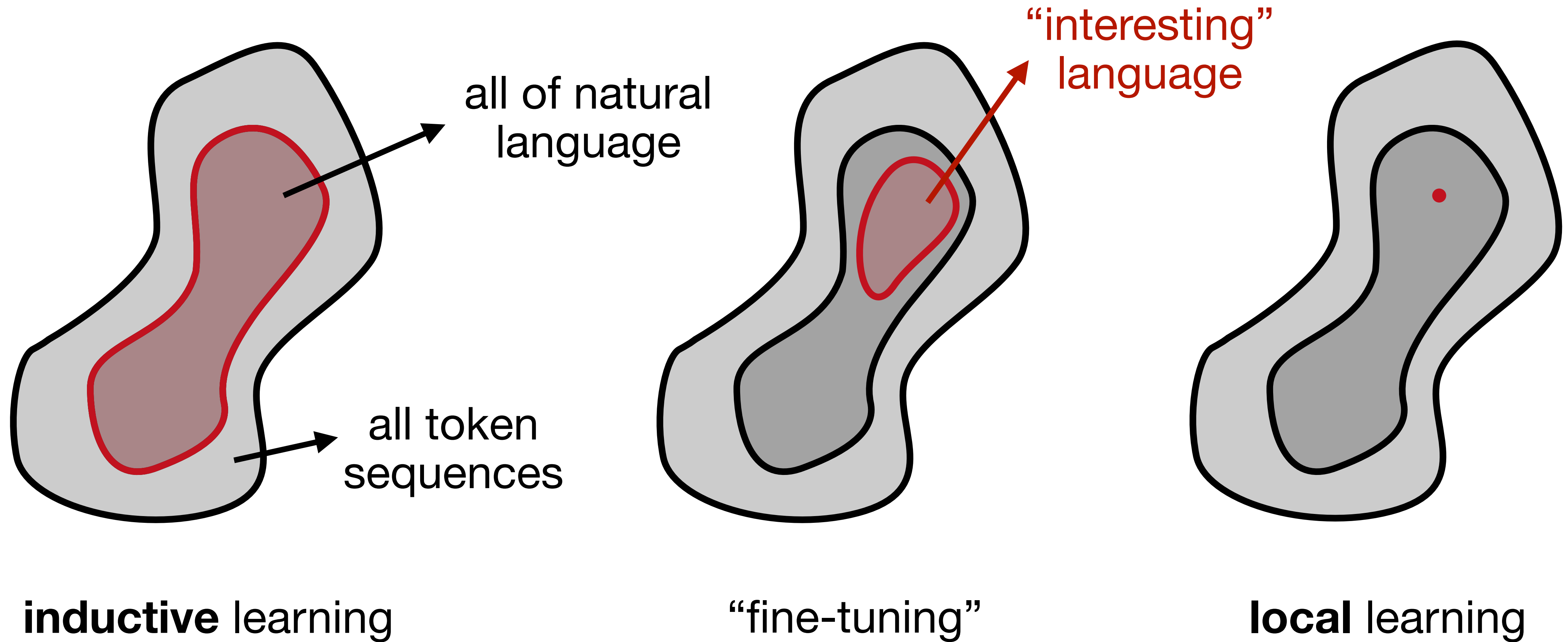
→ a small model class can fit a rich function class!

→ one local model needs only little data!

→ too good to be true?



# Local learning in a picture





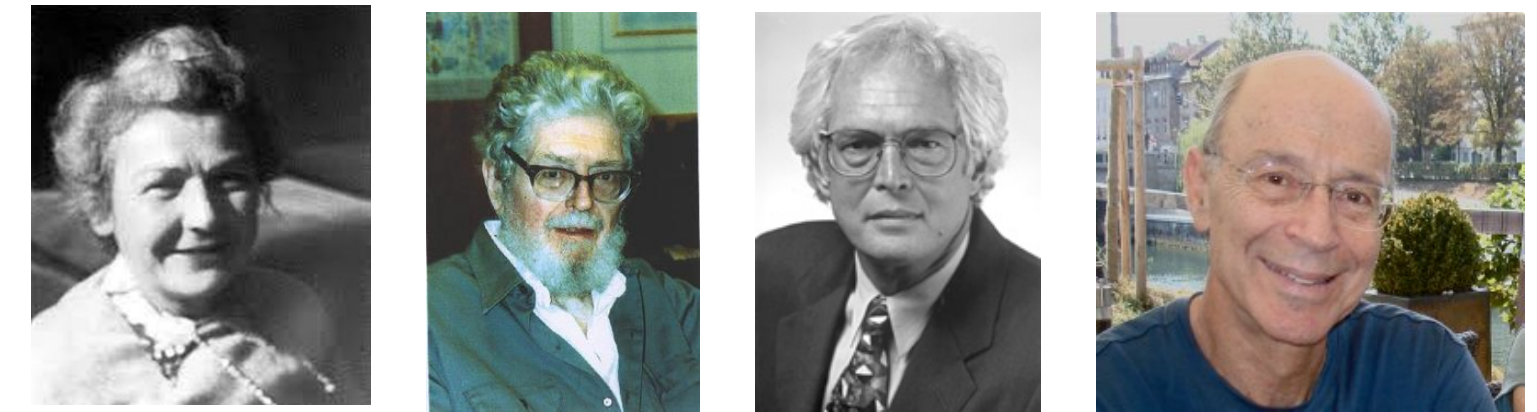
# History

- **since 1950s:** k-nearest neighbors
- **since 1960s:** kernel regression
- **since 1970s:** local (linear) learning
- **since 1980s:** transductive learning

“When solving a problem of interest, do not solve a more general problem as an intermediate step. Try to get the answer that you really need but not a more general one.”

**in 1990s:** local fine-tuning

CNNs on MNIST



Fix

Hodges

Cover

Hart

(Nadaraya & Watson)

(Cleveland & Devlin)

(Vapnik)

(Vapnik & Bottou)

# History

● **since 2020s:** (few-shot) in-context learning

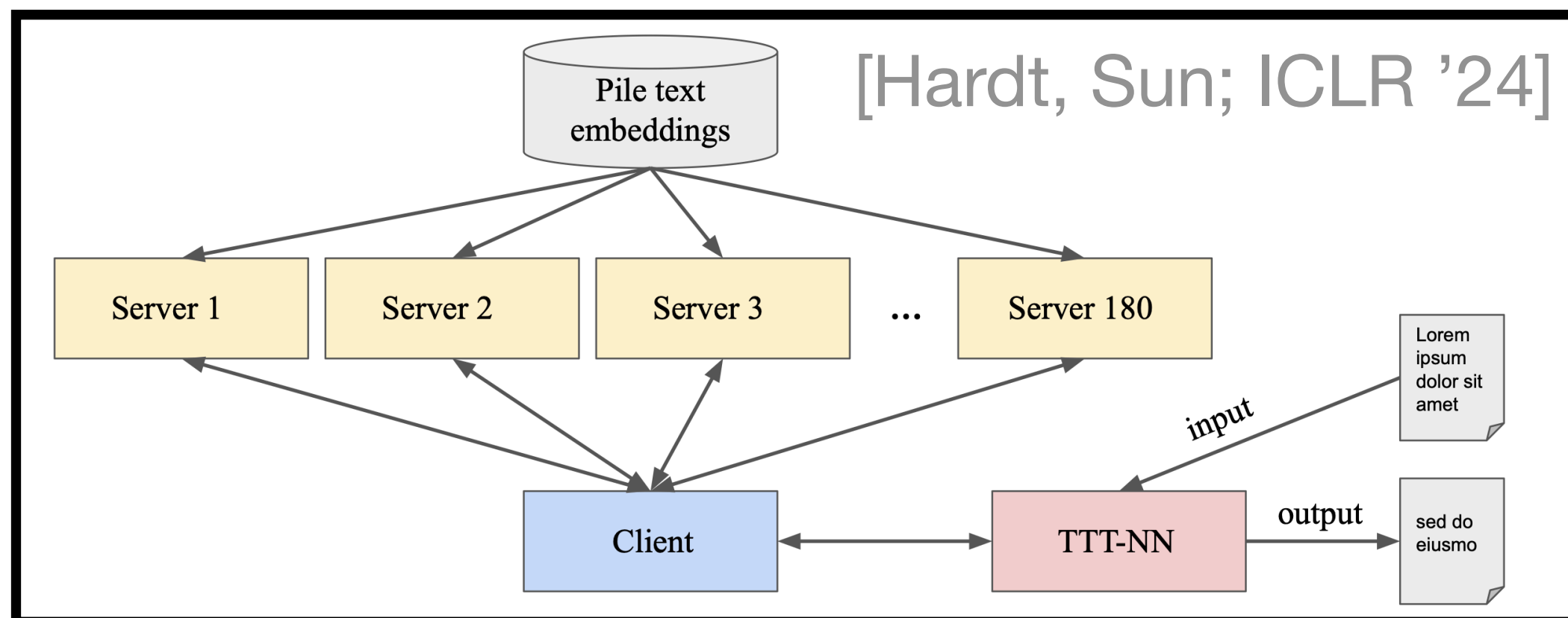
(GPT-3)

parametric controller: LLM

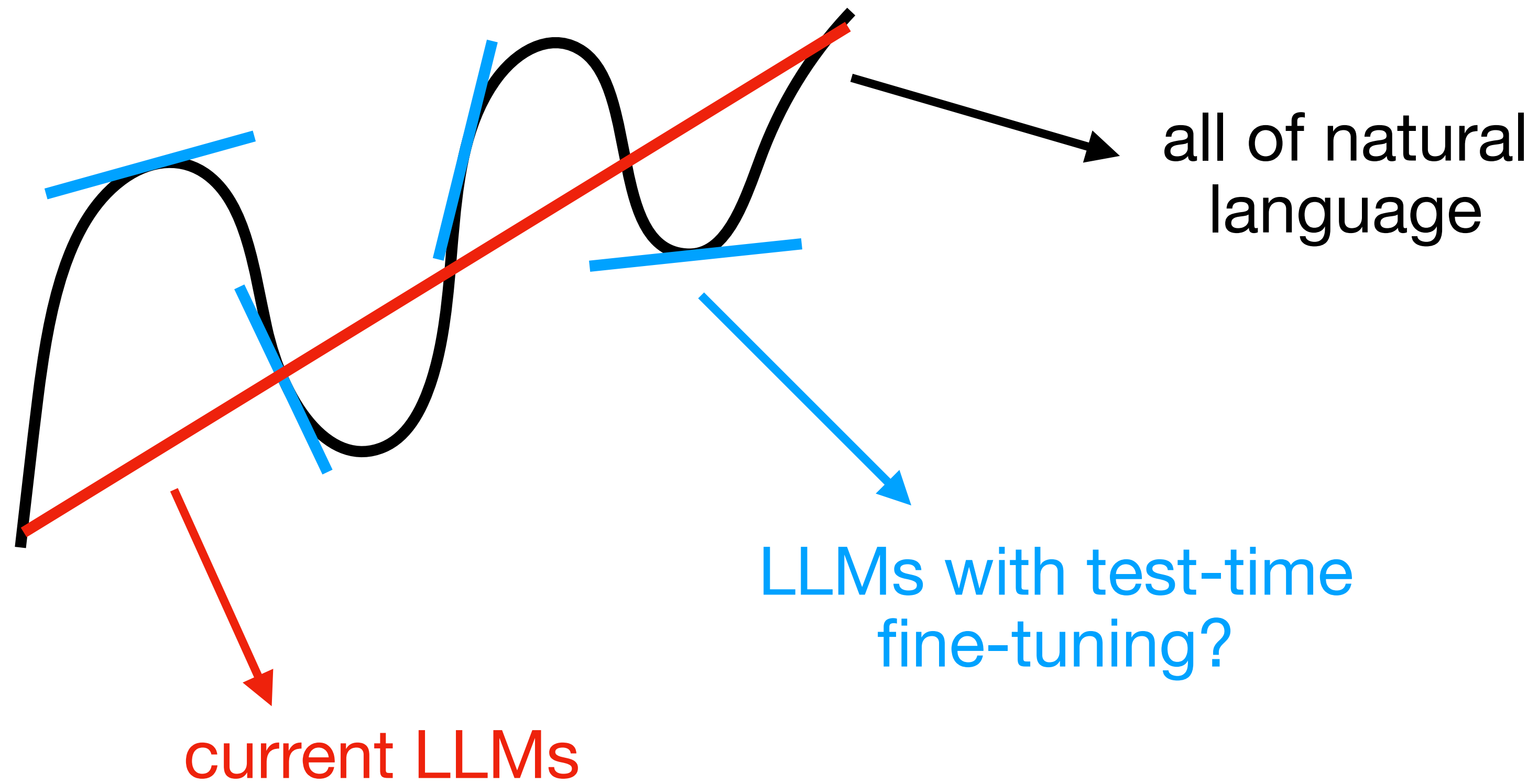
non-parametric memory: context (+ retrieval from database)

● **recently:** local fine-tuning (again!) with GPT-2

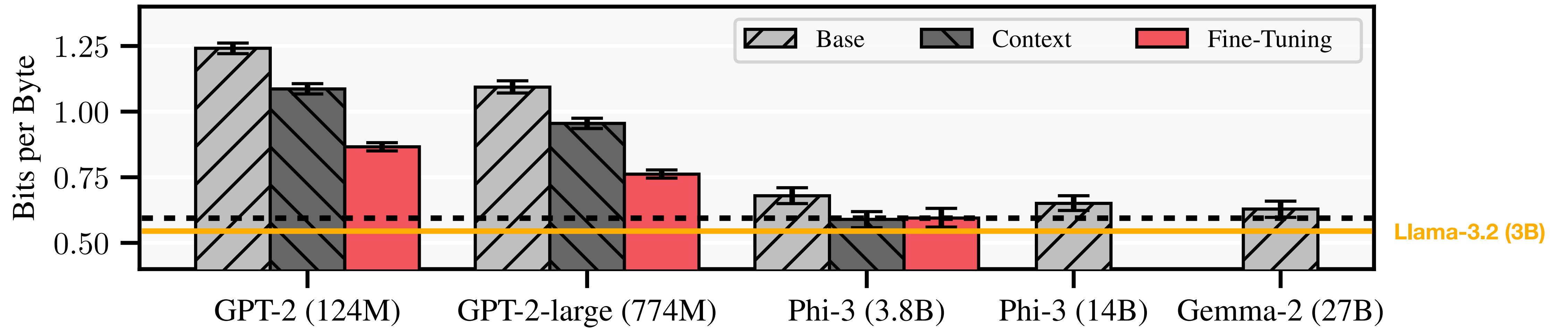
(Hardt & Sun)



# Hypothesis for LLMs



# Does local learning work with LLMs?



	Context	Fine-Tuning	$\Delta$
GitHub	74.6 (2.5)	<b>28.6</b> (2.2)	↓56.0
DeepMind Math	100.2 (0.1)	<b>70.1</b> (2.1)	↓30.1
US Patents	87.4 (2.5)	<b>62.2</b> (3.6)	↓25.2
FreeLaw	87.2 (3.6)	<b>65.5</b> (4.2)	↓21.7

GPT-2

	Context	Fine-Tuning	$\Delta$
GitHub	74.6 (2.5)	<b>31.0</b> (2.2)	↓43.6
DeepMind Math	100.2 (0.7)	<b>74.2</b> (2.3)	↓26.0
US Patents	87.4 (2.5)	<b>64.7</b> (3.8)	↓22.7
FreeLaw	87.2 (3.6)	<b>68.3</b> (4.2)	↓18.9

GPT-2-large

	Context	Fine-Tuning	$\Delta$
DeepMind Math	100.8	75.3	↓25.5
GitHub	71.3	46.5	↓24.8
FreeLaw	78.2	67.2	↓11.0
ArXiv	101.0	94.3	↓6.4

Phi-3

**Key challenge: which data to select?**

# Key challenge: which data to select?

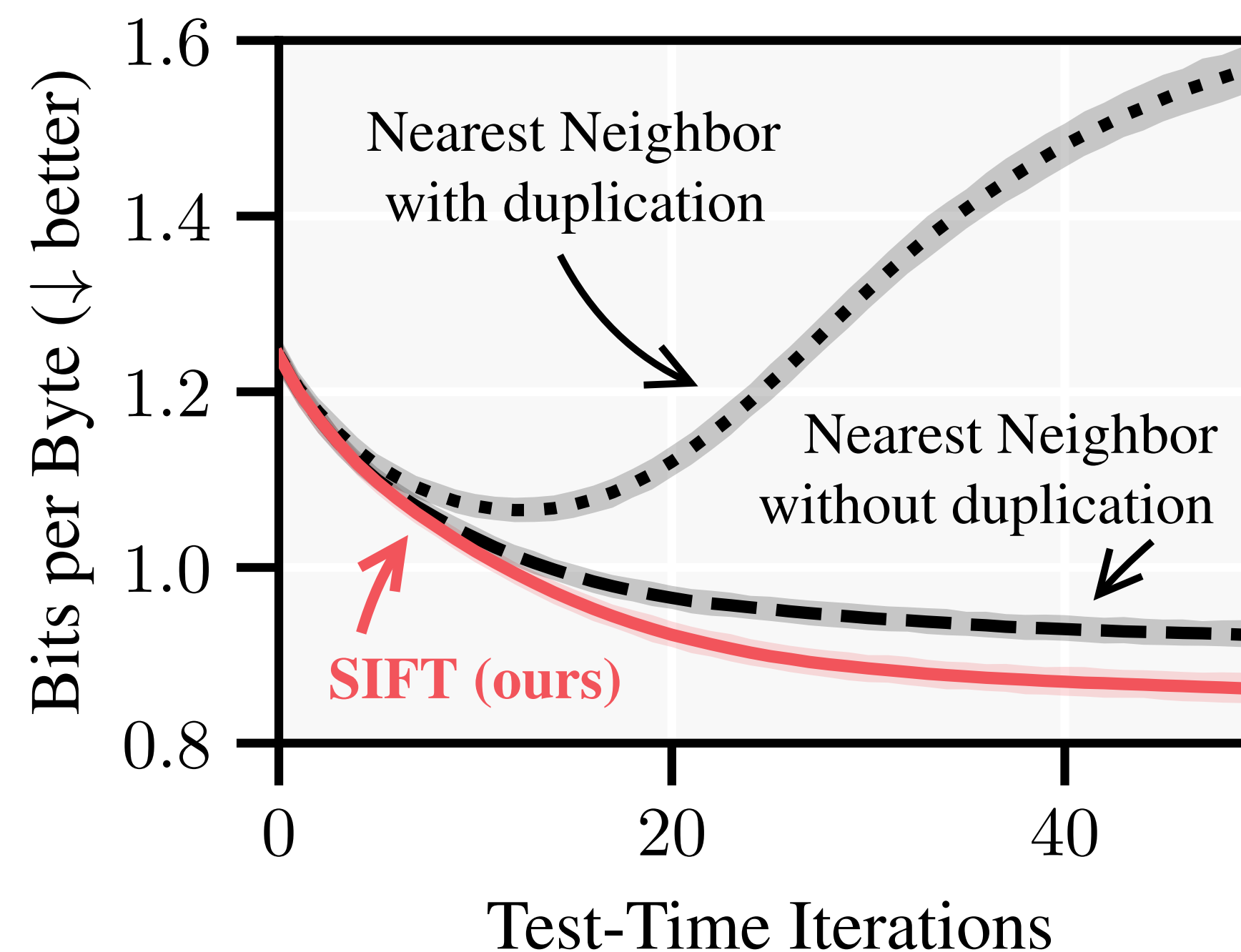
**Prompt:** What is the age of Michael Jordan and **how many kids does he have?**

**Nearest Neighbor:**

1. The age of Michael Jordan is 61 years.
2. Michael Jordan was born on February 17, 1963.

**SIFT (ours):**

1. The age of Michael Jordan is 61 years.
2. **Michael Jordan has five children.**



# Key challenge: which data to select?

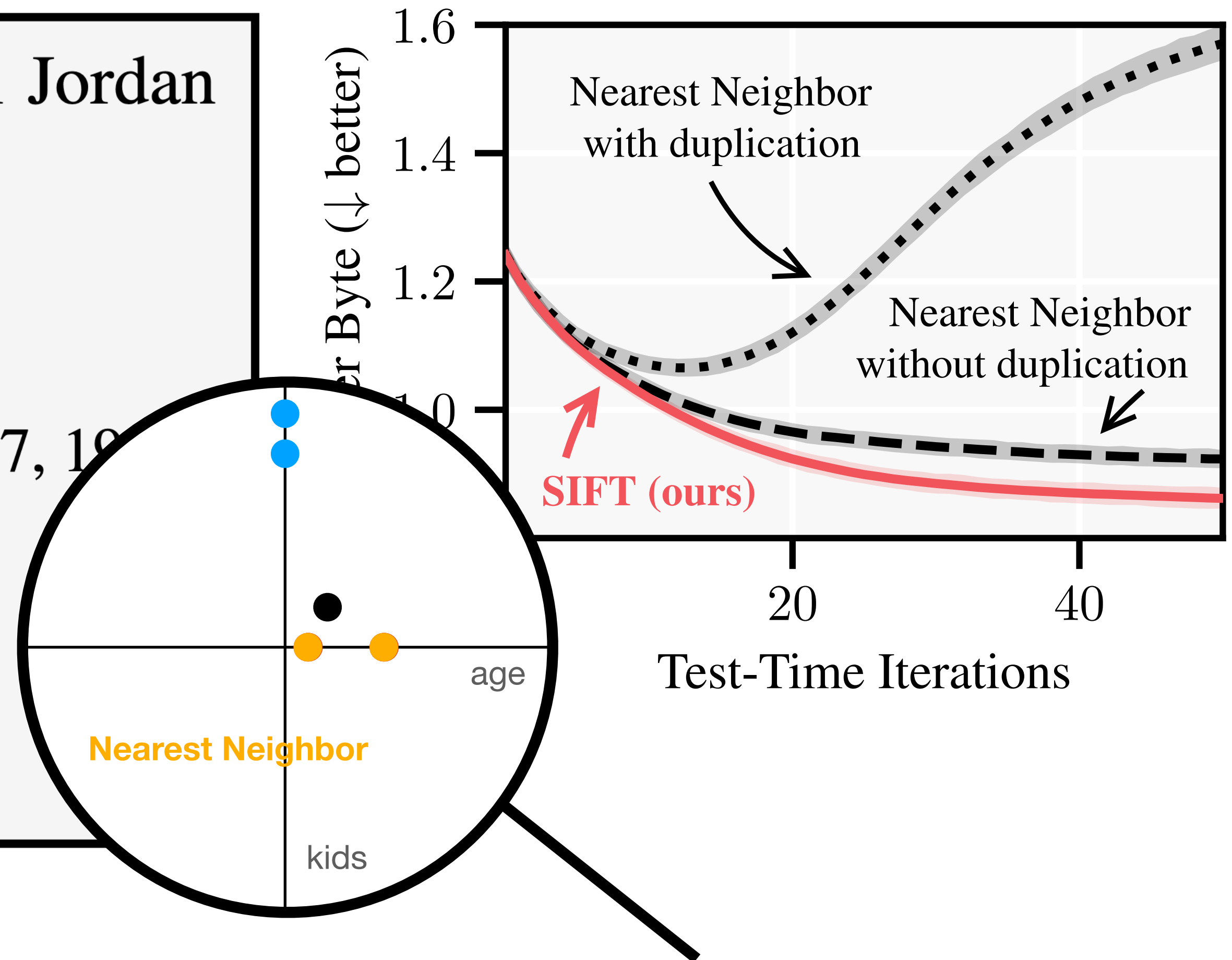
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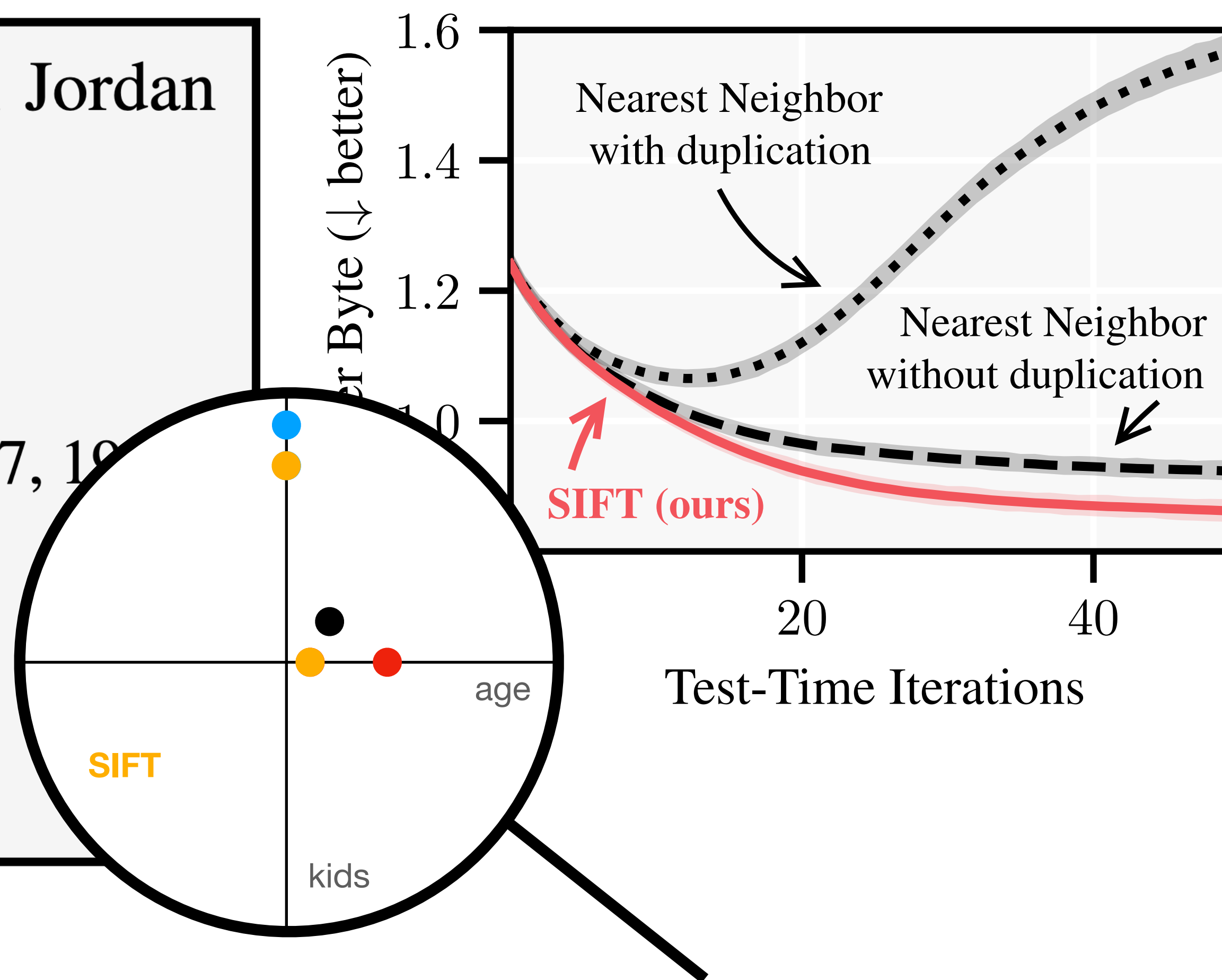
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# SIFT: selecting informative data for fine-tuning

## Principle:

Select data that *maximally* reduces “uncertainty” about how to respond to the prompt.

1. Estimate uncertainty
2. Minimize “posterior” uncertainty

[H, Bongni, Hakimi, Krause; preprint]

# 1 Estimating uncertainty

- Making this tractable:

Surrogate model: logit-linear model  $s(f^\star(x))$  with  $f^\star(x) = \mathbf{W}^\star \boldsymbol{\phi}(x)$

→ linear representation hypothesis [Park, Choe, Veitch; ICML '24]

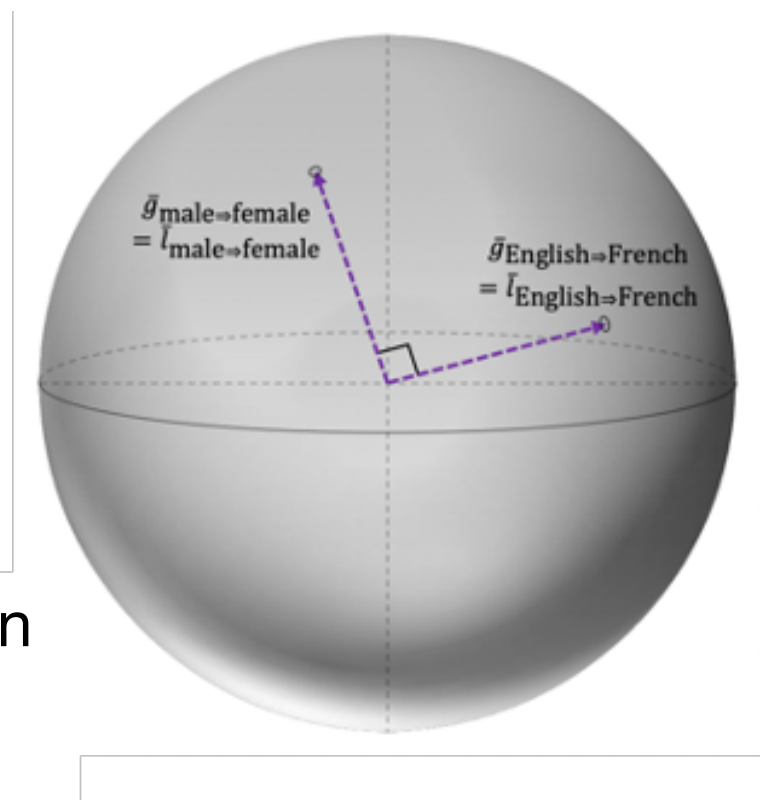
$$\mathcal{L}^\lambda(\mathbf{W}; D) = \underbrace{- \sum_{(x,y) \in D} \log s_y(f(x; \mathbf{W}))}_{\text{cross-entropy loss (NLL)}} + \underbrace{\frac{\lambda}{2} \|\mathbf{W} - \mathbf{W}^{\text{pre}}\|_F^2}_{\text{regularization}} \quad \mathbf{W}_n = \arg \min_{\mathbf{W}} \mathcal{L}^\lambda(\mathbf{W}; D_n)$$

$$s^\star(x) = s(f^\star(x)) \quad s_n(x) = s(\mathbf{W}_n \boldsymbol{\phi}(x))$$

"truth"                      model trained on  $n$  pieces of data

- Confidence sets:  $d_{\text{TV}}(s_n(x), s^\star(x)) \leq \beta_n(\delta) \sigma_n(x)$  (w.p.  $1 - \delta$ )
- error                      scaling      key object

→  $\sigma_n(x)$  measures **uncertainty** about response to  $x$ !



# 1 Estimating uncertainty

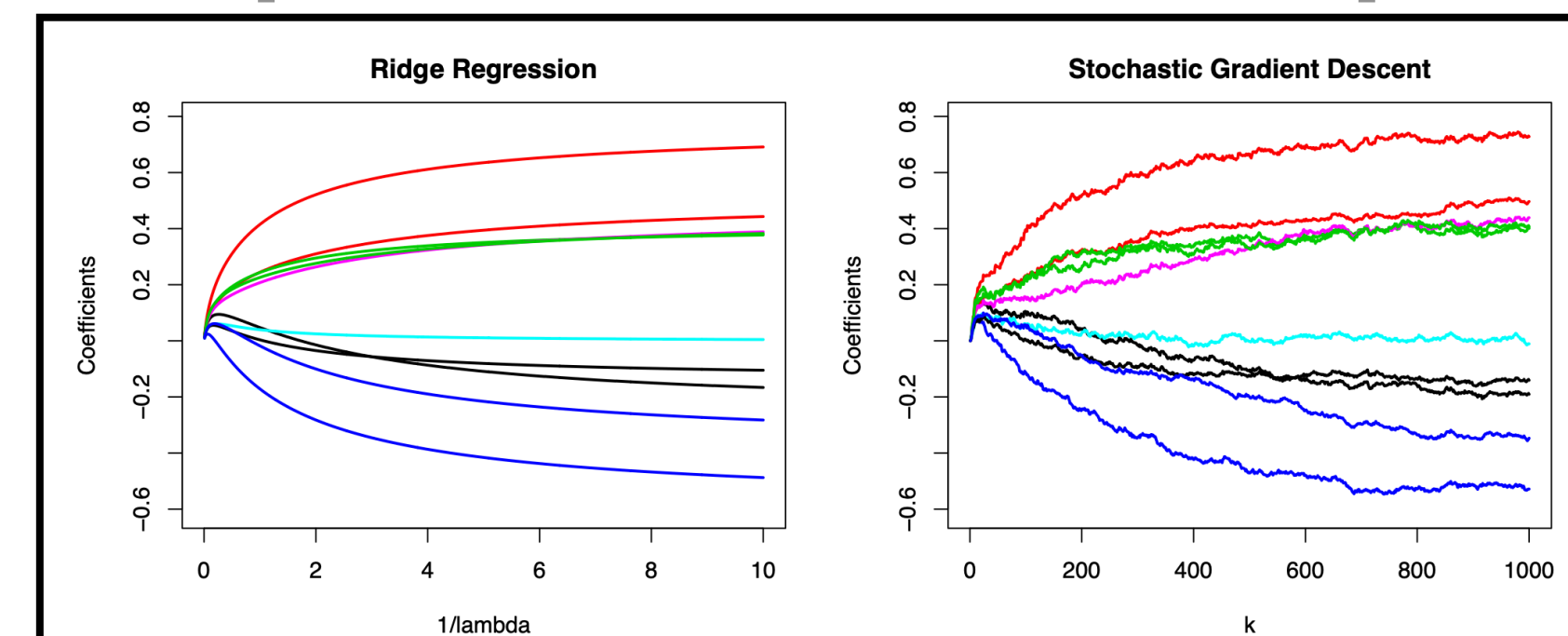
- Are **regularized loss minimization** and **fine-tuning** related?

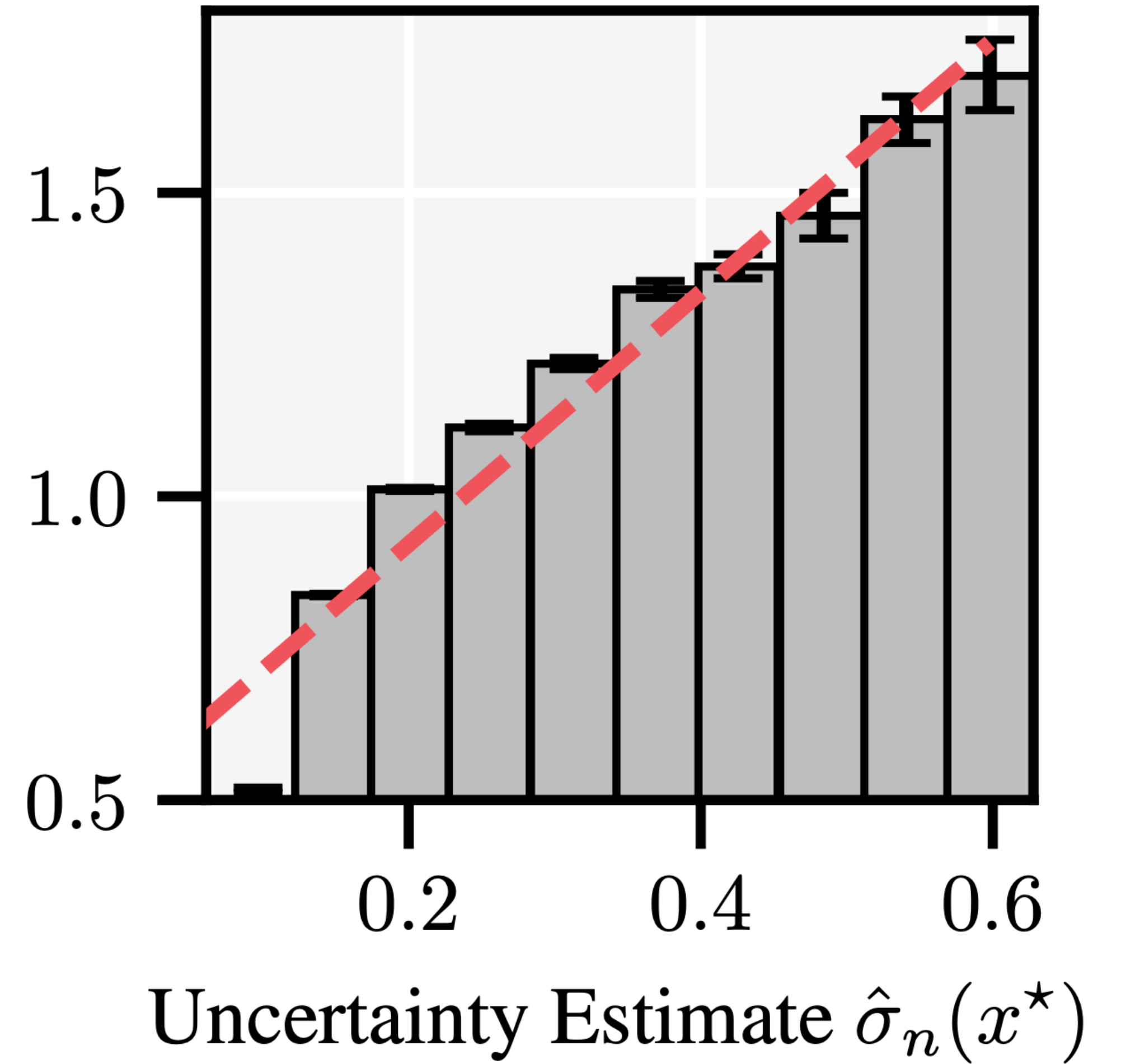
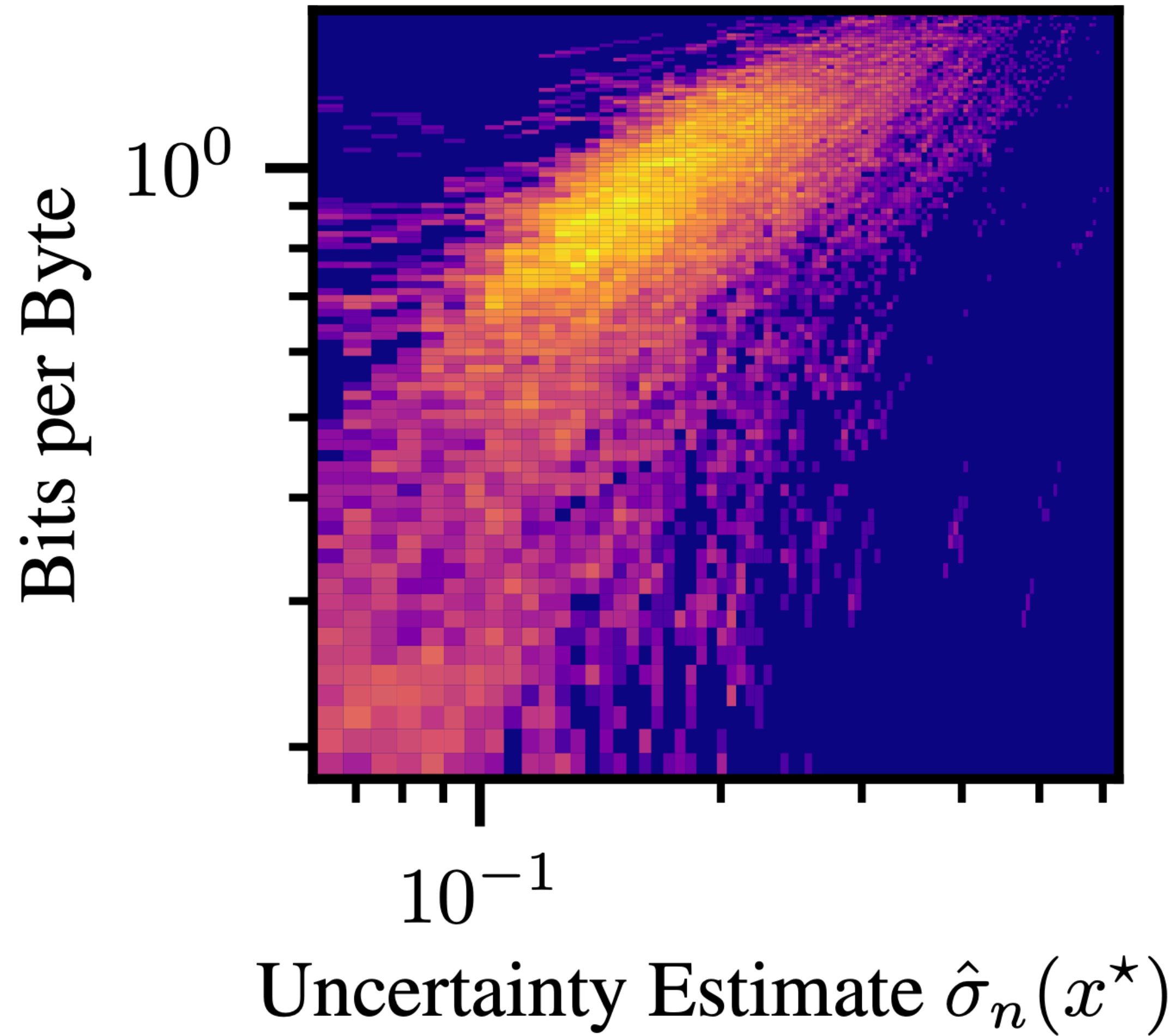
Consider two alternative models:

- $W_\lambda = \arg \min_W \mathcal{L}^\lambda(W)$  → minimizer of regularized loss
- $\widehat{W}_\eta = W^{\text{pre}} - \eta \nabla \mathcal{L}(W^{\text{pre}})$  → single gradient-step fine-tuning ( $\mathcal{L}$  is NLL)
- *Proposition:*  $\|W_{1/\eta} - \widehat{W}_\eta\|_F \leq \eta \|\nabla \mathcal{L}(W_{1/\eta}) - \nabla \mathcal{L}(W^{\text{pre}})\|$

[see also Ali et al.; ICML '20]

→ models similar for  $\lambda \approx 1/\eta$ !





→  $\sigma_n(x)$  measures **uncertainty** about response to  $x$ !

## ② Minimizing “posterior” uncertainty

- Choose data that minimizes uncertainty of the model after seeing this data:

$$\begin{aligned}
 x_{n+1} &= \operatorname{argmin}_x \sigma_{X_n \cup \{x\}}(x^\star) \quad \leftarrow \text{prompt} \\
 &= \operatorname{argmax}_x \begin{bmatrix} k(x^\star, x_1) \\ \vdots \\ k(x^\star, x_n) \\ k(x^\star, x) \end{bmatrix}^\top \left( \begin{bmatrix} k(x_1, x_1) & \dots & k(x_1, x_n) & k(x_1, x) \\ \vdots & \ddots & \vdots & \vdots \\ k(x_n, x_1) & \dots & k(x_n, x_n) & k(x_n, x) \\ k(x, x_1) & \dots & k(x, x_n) & k(x, x) \end{bmatrix} + \lambda_{n+1} \right)^{-1} \begin{bmatrix} k(x^\star, x_1) \\ \vdots \\ k(x^\star, x_n) \\ k(x^\star, x) \end{bmatrix} \quad \text{with } k(x, x') = \boldsymbol{\phi}(x)^\top \boldsymbol{\phi}(x') \\
 &\quad \text{maximize relevance} \qquad \qquad \qquad \text{minimize redundancy}
 \end{aligned}$$

- Convergence guarantee* (in case of no synergies):

$$\sigma_n^2(x^\star) - \underbrace{\sigma_\infty^2(x^\star)}_{\text{irreducible uncertainty}} \leq O(\lambda \log n) / \sqrt{n}$$

Not possible with nearest neighbor retrieval!

→ predictions can be only as good as the data and the learned abstractions!

## ② Minimizing “posterior” uncertainty (example)

- Example: suppose embeddings are normalized

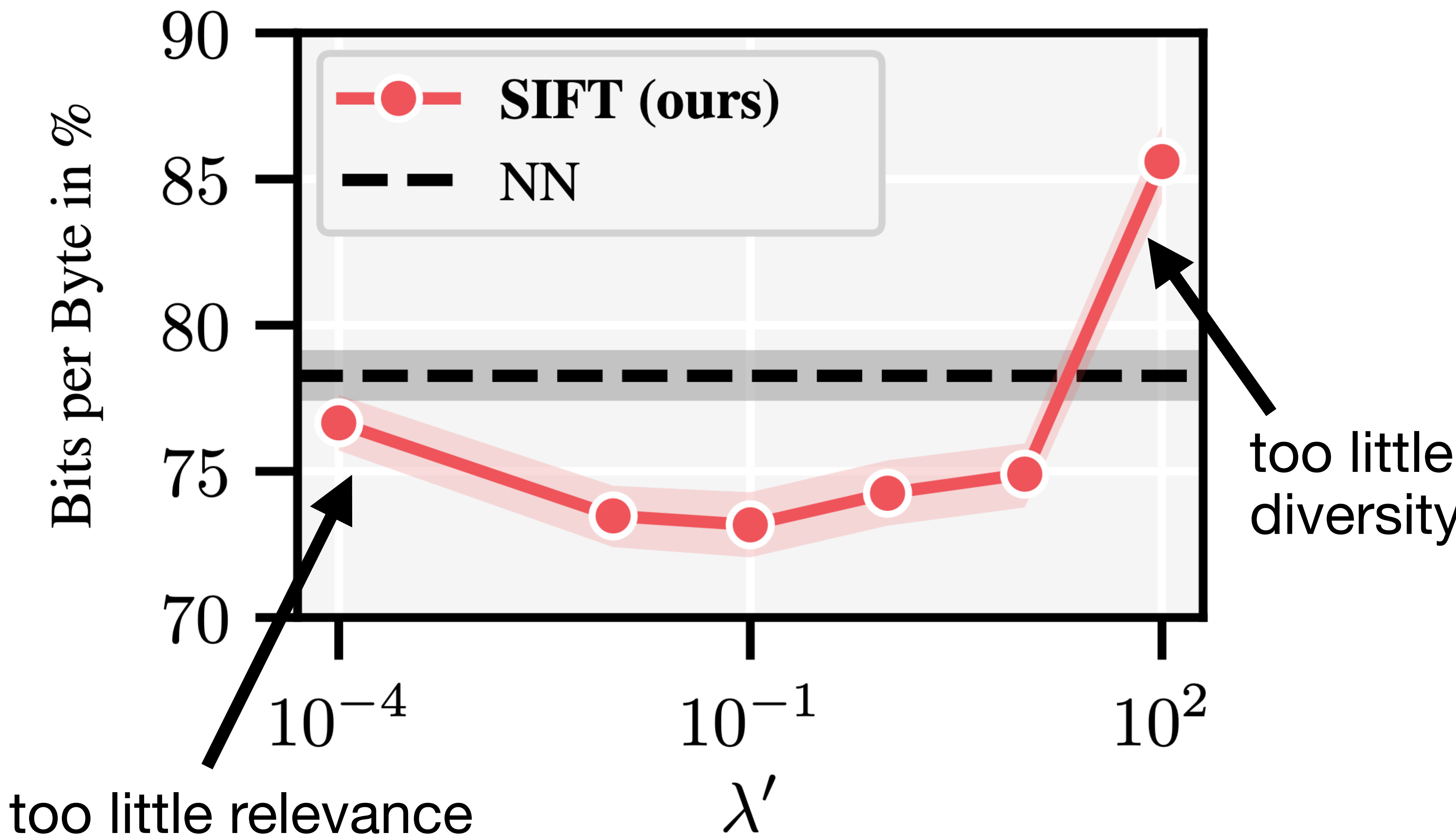
$$\mathbf{x}_1 = \arg \min_{\mathbf{x} \in \mathcal{D}} \sigma_{\{\mathbf{x}\}}^2(\mathbf{x}^*) = \arg \max_{\mathbf{x} \in \mathcal{D}} \frac{(\phi(\mathbf{x}^*)^\top \phi(\mathbf{x}))^2}{1 + \lambda} = \arg \max_{\mathbf{x} \in \mathcal{D}} \underbrace{\left( \angle_{\phi}(\mathbf{x}^*, \mathbf{x}) \right)^2}_{\text{cosine similarity of } \phi(\mathbf{x}^*), \phi(\mathbf{x})}. \quad \text{(1st point)}$$

$$\mathbf{x}_2 = \arg \min_{\mathbf{x} \in \mathcal{D}} \sigma_{\{\mathbf{x}_1, \mathbf{x}\}}^2(\mathbf{x}^*) = \arg \max_{\mathbf{x} \in \mathcal{D}} \begin{bmatrix} \angle_{\phi}(\mathbf{x}^*, \mathbf{x}_1) \\ \angle_{\phi}(\mathbf{x}^*, \mathbf{x}) \end{bmatrix}^\top \begin{bmatrix} 1 + \lambda & \angle_{\phi}(\mathbf{x}_1, \mathbf{x}) \\ \angle_{\phi}(\mathbf{x}_1, \mathbf{x}) & 1 + \lambda \end{bmatrix}^{-1} \begin{bmatrix} \angle_{\phi}(\mathbf{x}^*, \mathbf{x}_1) \\ \angle_{\phi}(\mathbf{x}^*, \mathbf{x}) \end{bmatrix}. \quad \text{(2nd point)}$$

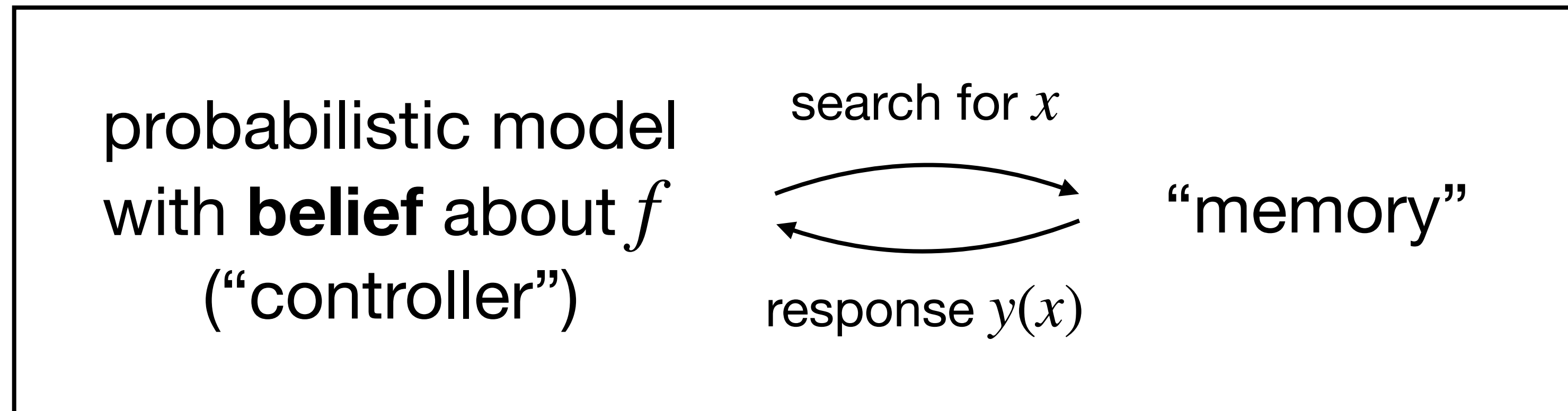
- Example: suppose  $\mathbf{x}$  is such that  $\angle_{\phi}(\mathbf{x}_1, \mathbf{x}) = 0$ . Then  $\mathbf{x}$  is preferred over  $\mathbf{x}_1$  iff

$$\angle_{\phi}(\mathbf{x}^*, \mathbf{x})^2 > \frac{\lambda}{2 + \lambda} \angle_{\phi}(\mathbf{x}^*, \mathbf{x}_1)^2$$

→ as  $\lambda \rightarrow \infty$ : **maximum relevance**, as  $\lambda \rightarrow 0$ : **minimum redundancy**



# A probabilistic interpretation of SIFT



Tractable Probabilistic Model

$$y(x) = f(x) + \varepsilon(x)$$

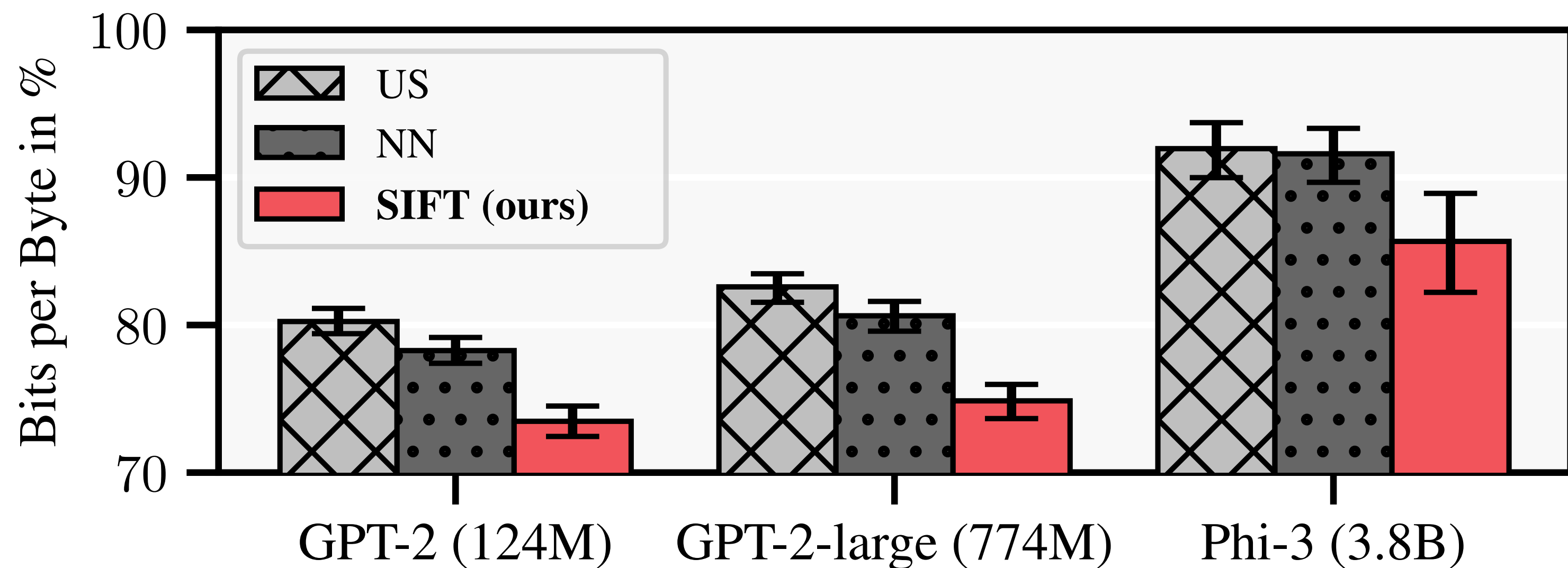
$$f \sim \mathcal{GP}(\mu, k)$$

$$\varepsilon(x) \stackrel{iid}{\sim} \mathcal{N}(0, \sqrt{\lambda})$$

$$\begin{aligned} x_{n+1} &= \arg \min_x \text{posterior variance } \sigma_n^2(x^\star) \text{ Var}(f(x^\star) \mid y_{1:n}, y(x)) \\ &= \arg \max_x \text{I}(f(x^\star); y(x) \mid y_{1:n}) \\ &= \arg \max_x \underbrace{\text{I}(f(x^\star); y(x))}_{\text{relevance}} - \underbrace{\text{I}(f(x^\star); y(x); y_{1:n})}_{\text{redundancy}} \end{aligned}$$



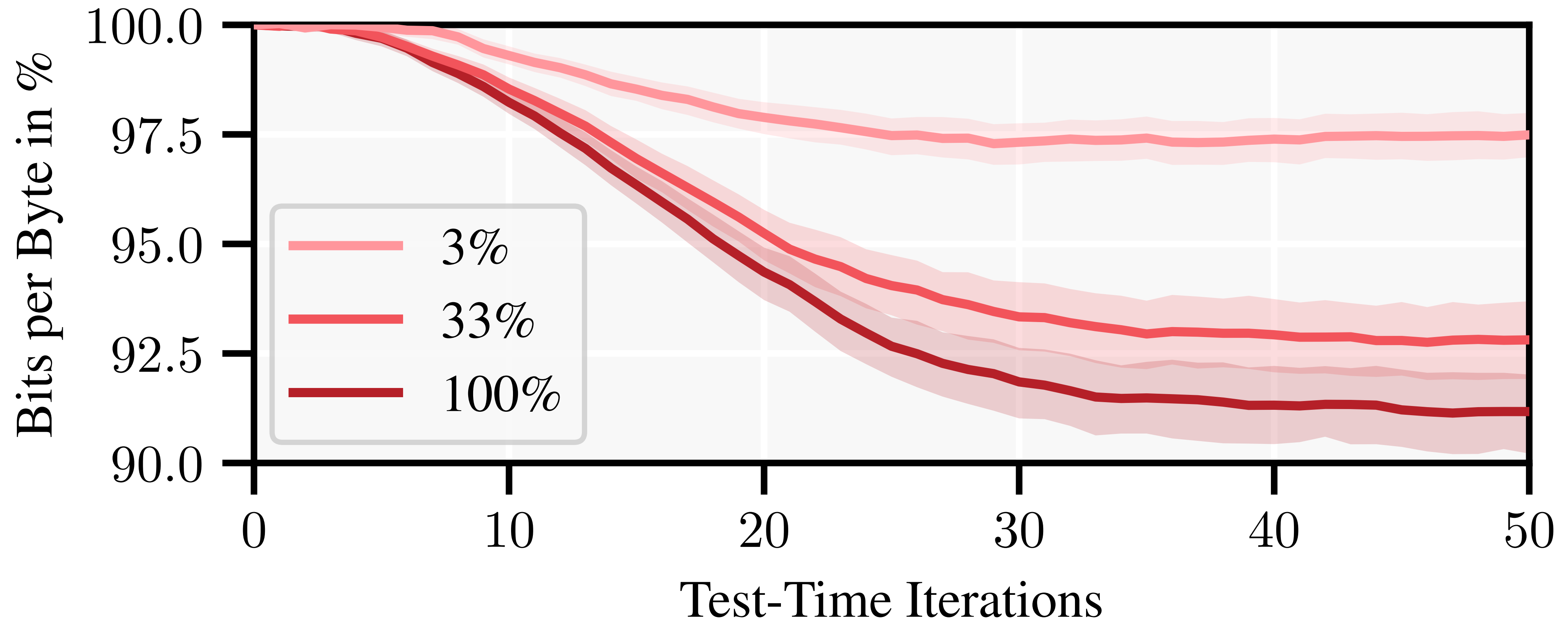
# Does SIFT work?



	US	NN	NN-F	SIFT	$\Delta$
NIH Grants	93.1 (1.1)	84.9 (2.1)	91.6 (16.7)	<b>53.8</b> (8.9)	↓31.1
US Patents	85.6 (1.5)	80.3 (1.9)	108.8 (6.6)	<b>62.9</b> (3.5)	↓17.4
GitHub	45.6 (2.2)	42.1 (2.0)	53.2 (4.0)	<b>30.0</b> (2.2)	↓12.1
Enron Emails	<b>68.6</b> (9.8)	<b>64.4</b> (10.1)	91.6 (20.6)	<b>53.1</b> (11.4)	↓11.3
Wikipedia	67.5 (1.9)	<b>66.3</b> (2.0)	121.2 (3.5)	<b>62.7</b> (2.1)	↓3.6
Common Crawl	92.6 (0.4)	90.4 (0.5)	148.8 (1.5)	<b>87.5</b> (0.7)	↓2.9
PubMed Abstr.	88.9 (0.3)	87.2 (0.4)	162.6 (1.3)	<b>84.4</b> (0.6)	↓2.8
ArXiv	85.4 (1.2)	<b>85.0</b> (1.6)	166.8 (6.4)	<b>82.5</b> (1.4)	↓2.5
PubMed Central	<b>81.7</b> (2.6)	<b>81.7</b> (2.6)	155.6 (5.1)	<b>79.5</b> (2.6)	↓2.2
Stack Exchange	78.6 (0.7)	78.2 (0.7)	141.9 (1.5)	<b>76.7</b> (0.7)	↓1.5
Hacker News	<b>80.4</b> (2.5)	<b>79.2</b> (2.8)	133.1 (6.3)	<b>78.4</b> (2.8)	↓0.8
FreeLaw	<b>63.9</b> (4.1)	<b>64.1</b> (4.0)	122.4 (7.1)	<b>64.0</b> (4.1)	↑0.1
DeepMind Math	<b>69.4</b> (2.1)	<b>69.6</b> (2.1)	121.8 (3.1)	<b>69.7</b> (2.1)	↑0.3
<i>All</i>	80.2 (0.5)	78.3 (0.5)	133.3 (1.2)	<b>73.5</b> (0.6)	↓4.8

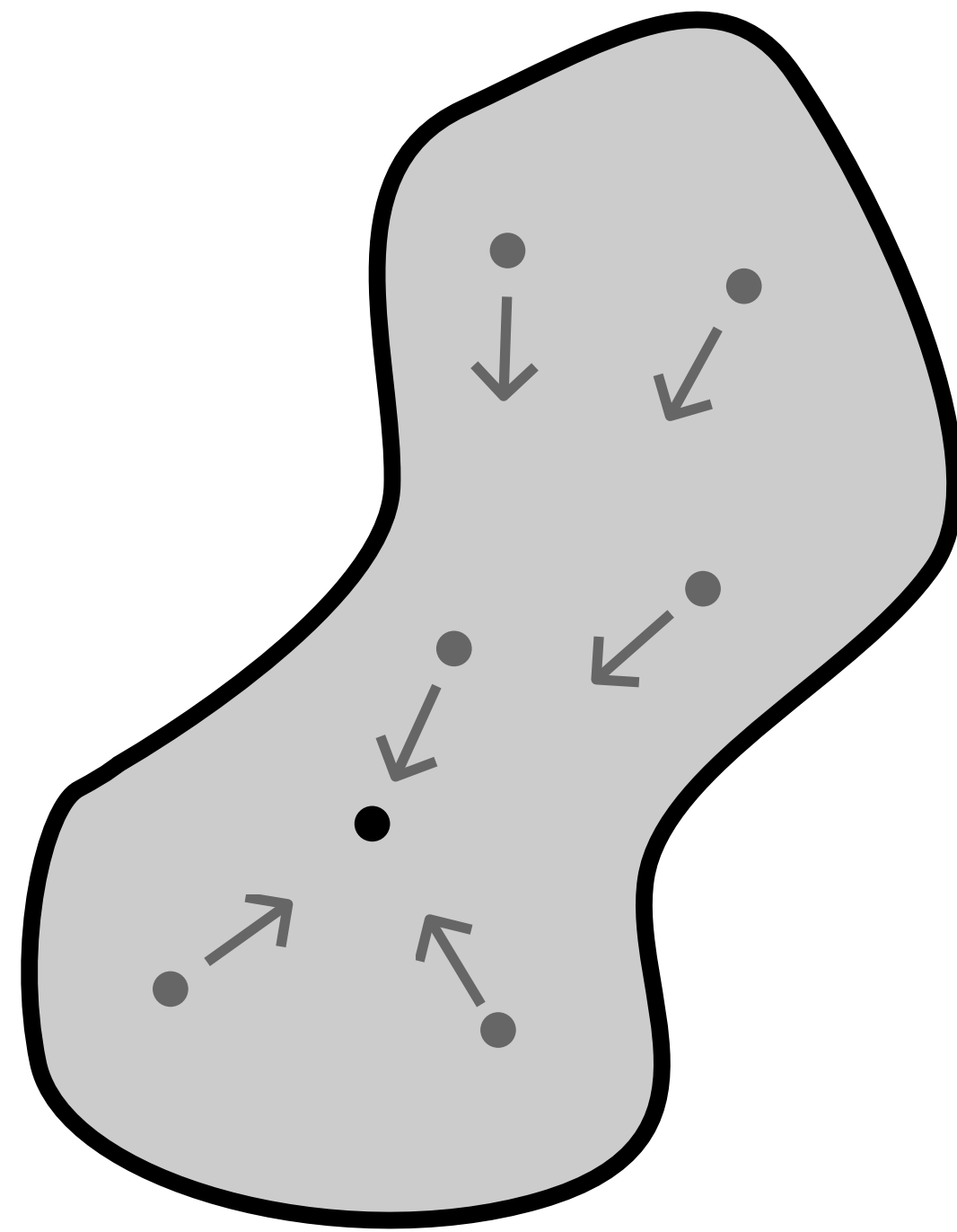
→ larger gains with stronger base models!

# Does SIFT work?



→ larger gains with larger “memory”!

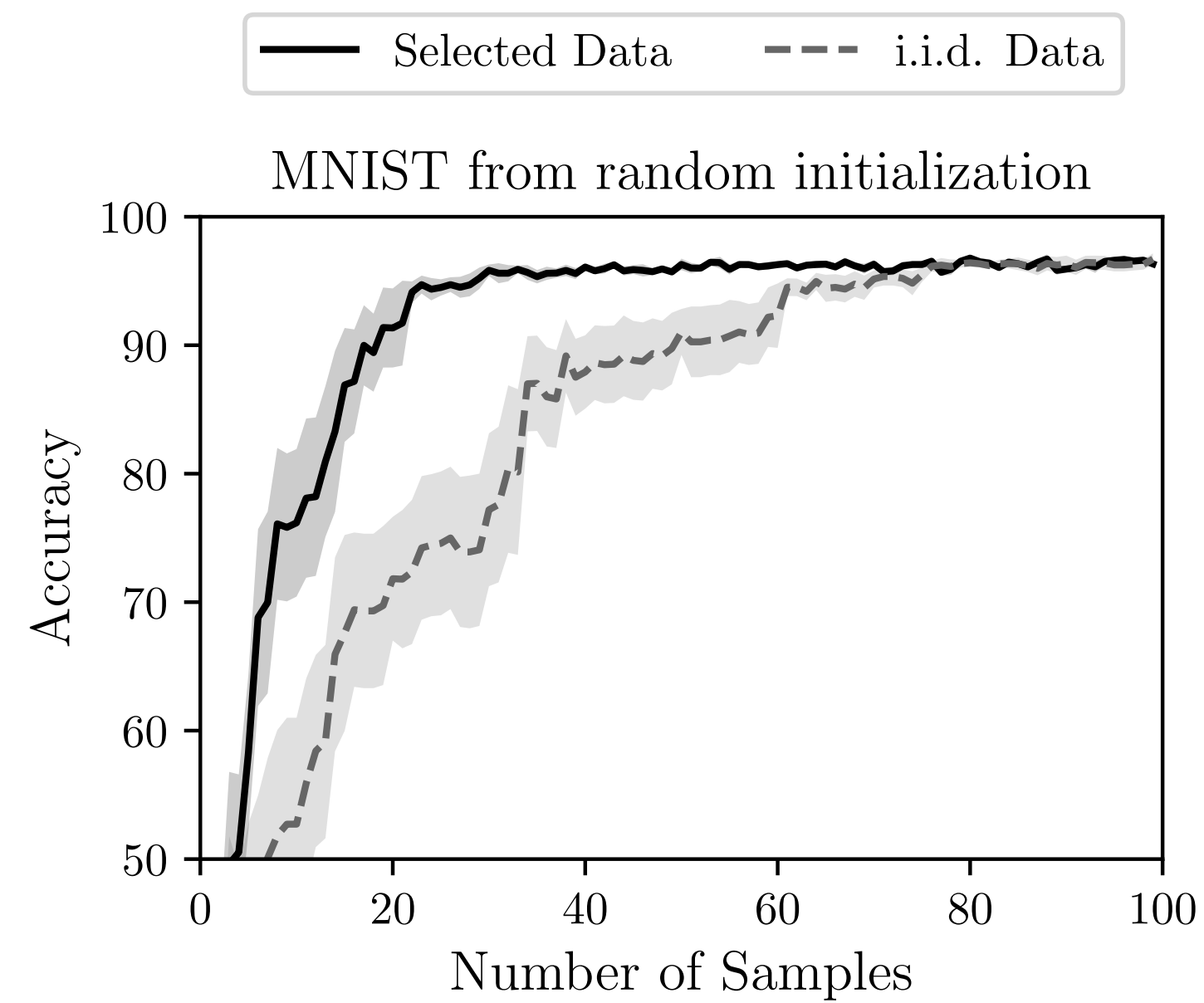
# Can we learn representations over time?



representations

Strong representations can be bootstrapped!

[H, Sukhija, Treven, As, Krause; NeurIPS '24]



# Summary

## **Local models**

solve one problem at a time

## **Inductive models** (most current SOTA models)

attempt to solve all possible problems at once

→ local learning allows allocating compute where it is “interesting”!

# I'm happy to chat!

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- **Transductive Active Learning: Theory and Applications**

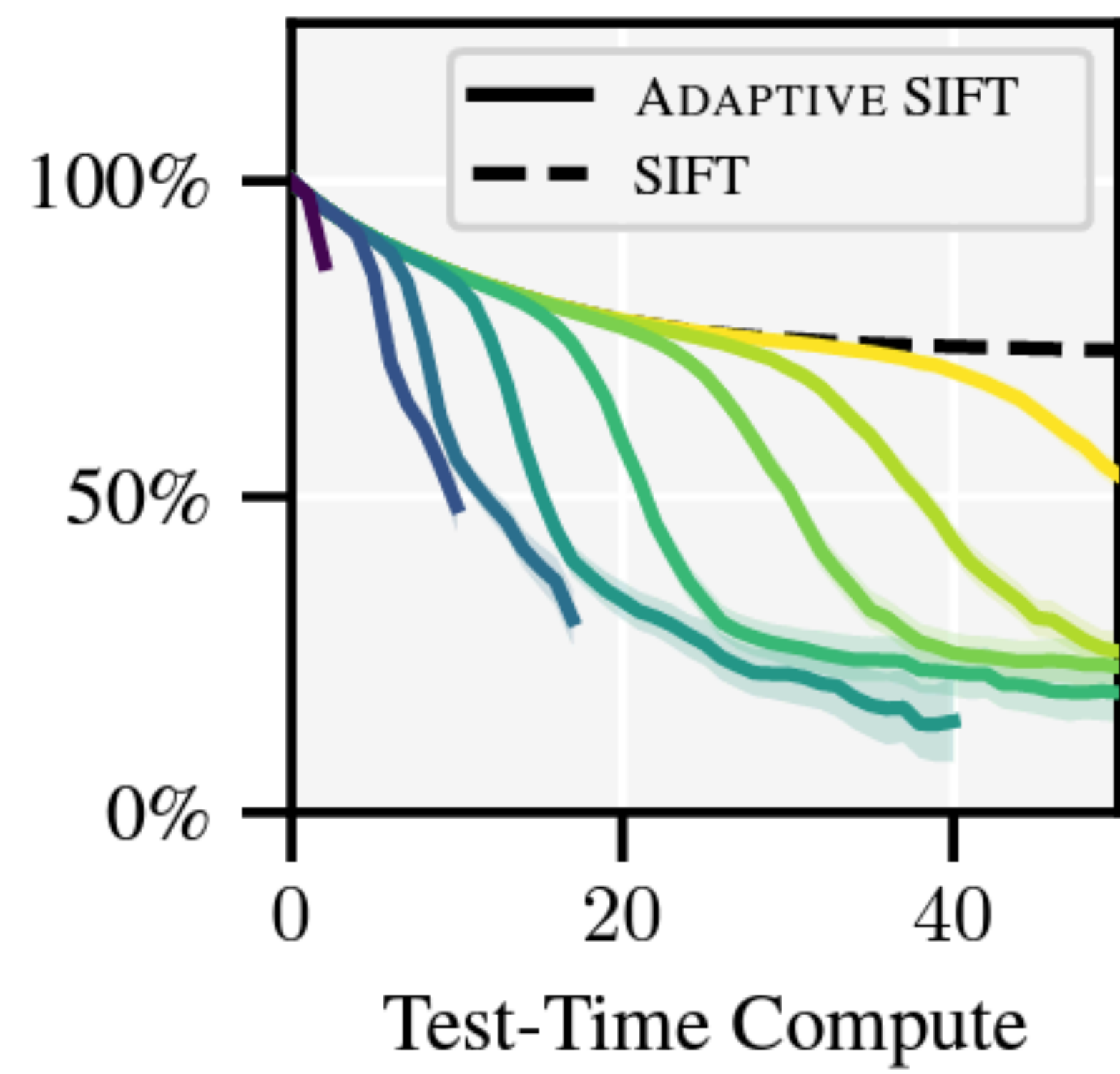
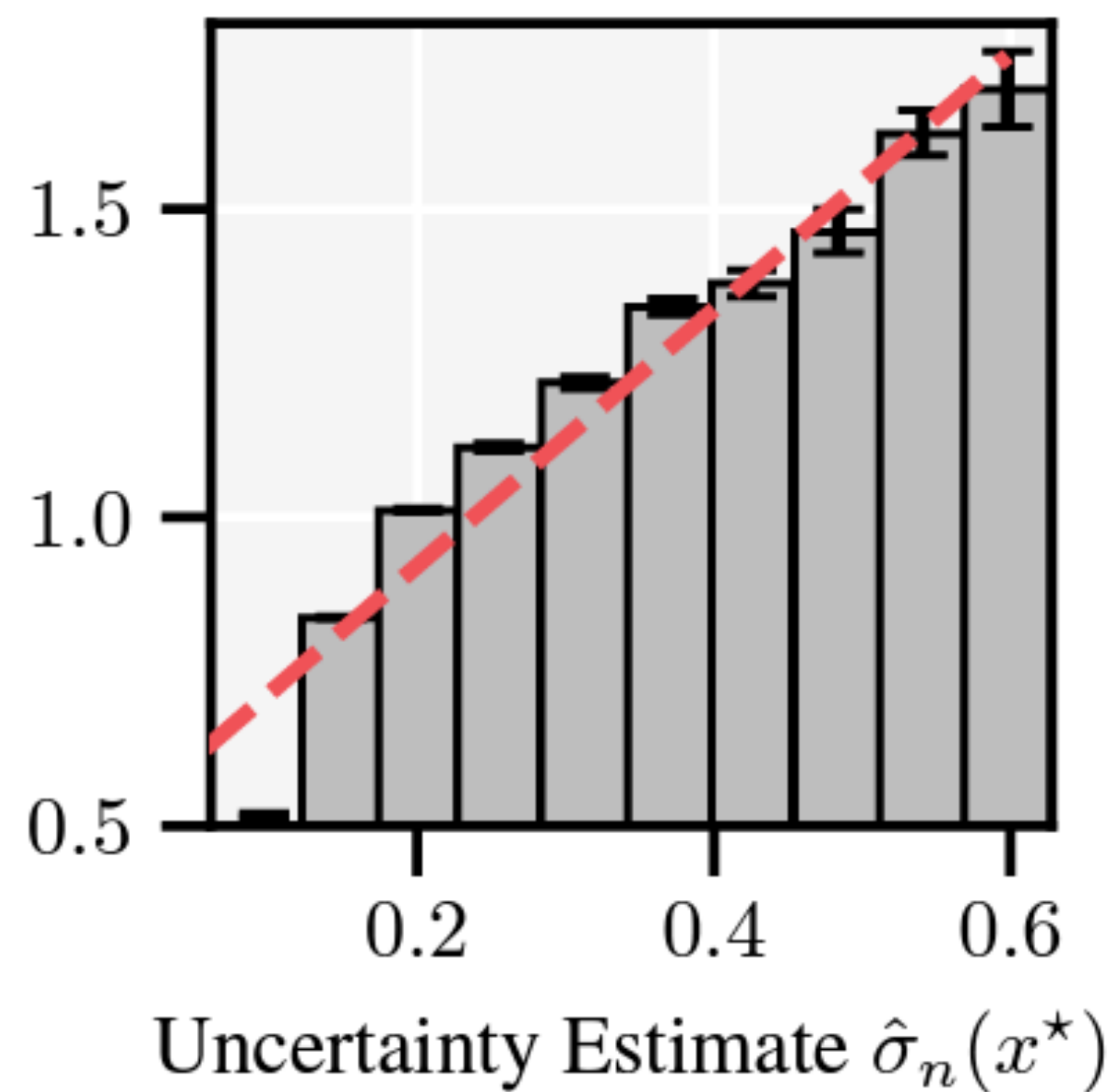
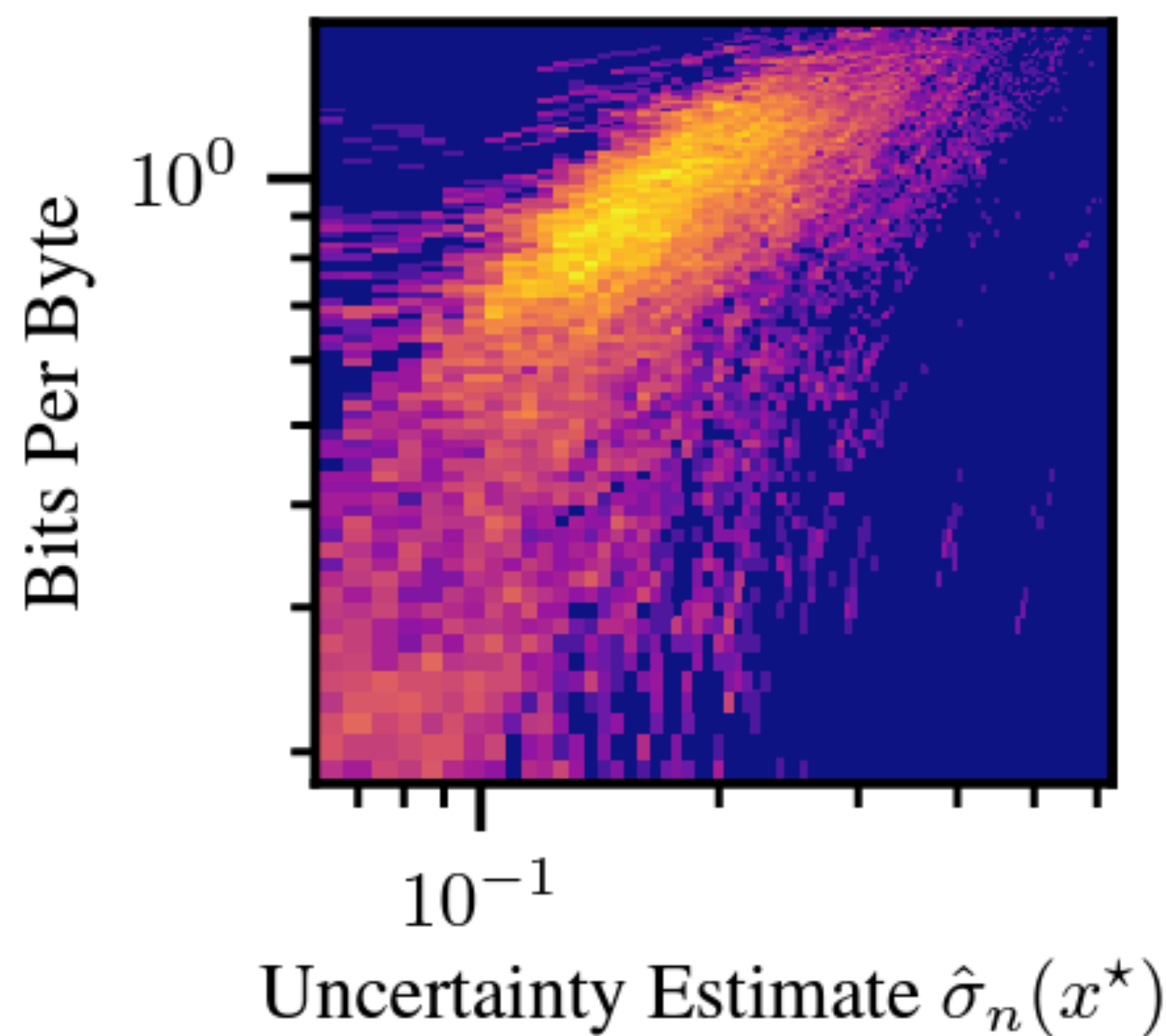
NeurIPS '24

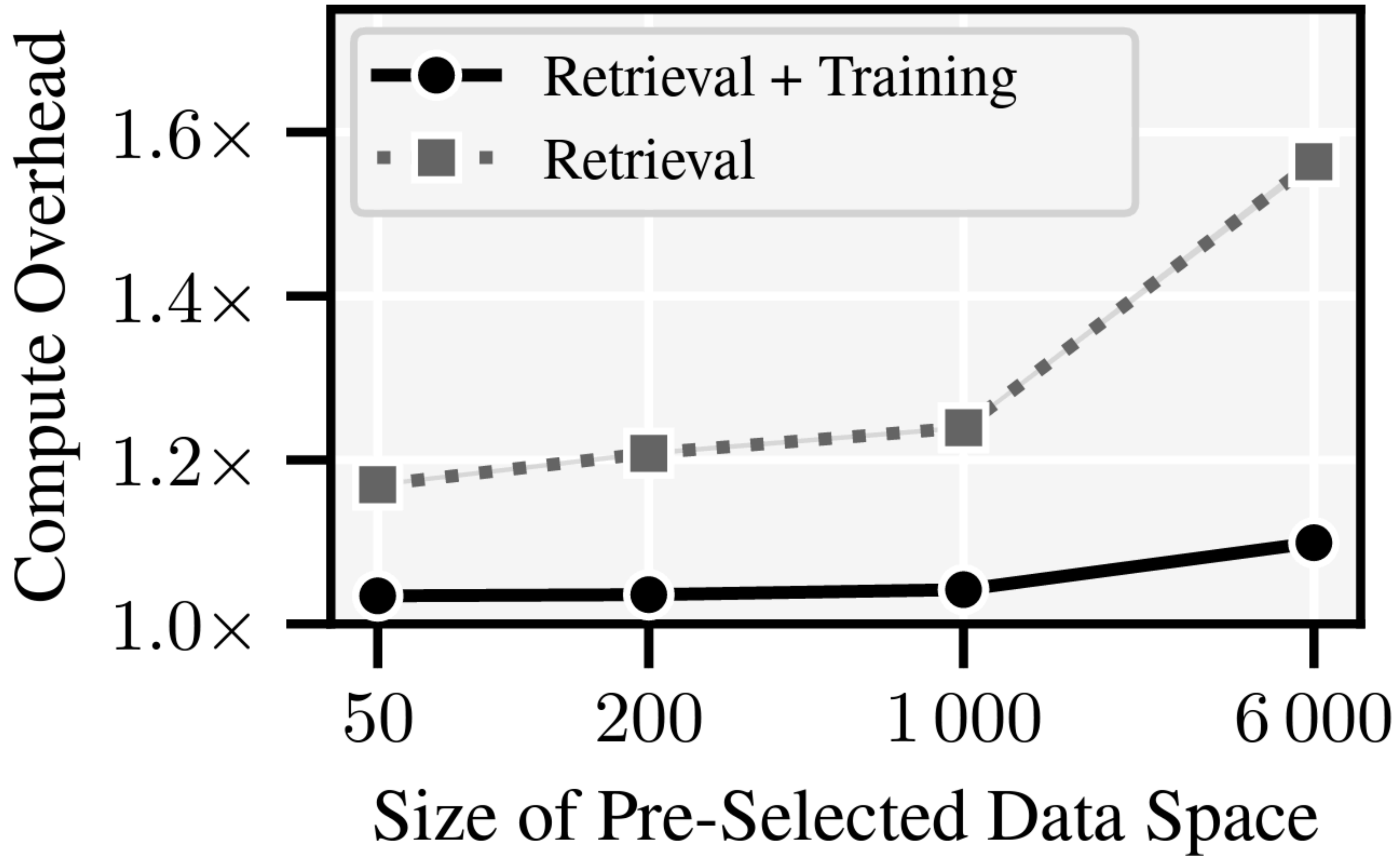


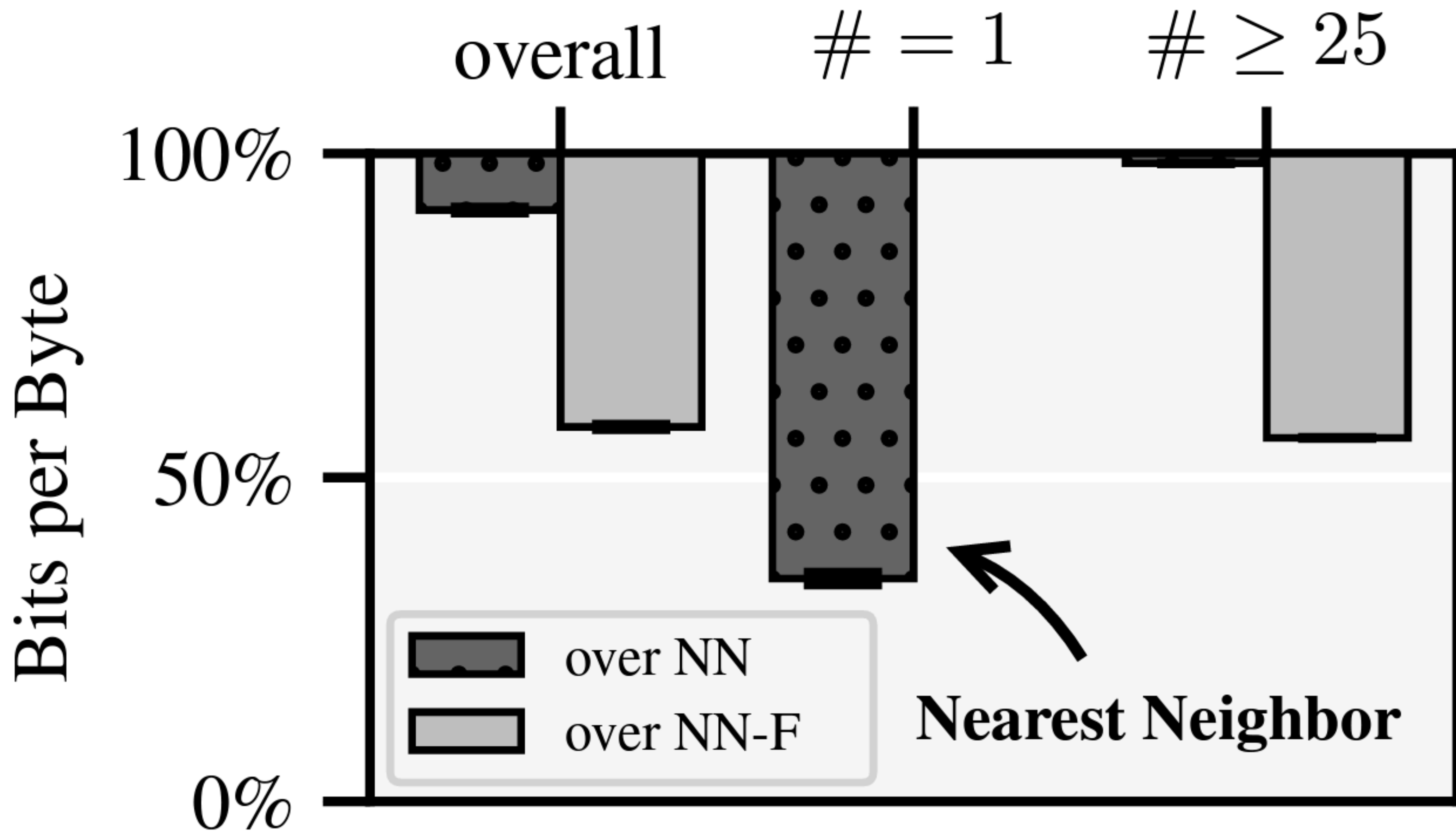
- **Efficiently Learning at Test-Time: Active Fine-Tuning of LLMs**

NeurIPS '24 Workshop











<b>Model</b>	<b>Bits per Byte</b>	<b>Bits per Byte (without Wikipedia)</b>
Jurassic-1 (178B, <a href="#">Lieber et al., 2021</a> )	n/a	0.601
GLM (130B, <a href="#">Zeng et al., 2022</a> )	n/a	0.622
GPT-2 (124M, <a href="#">Radford et al., 2019</a> )	1.241	
GPT-2 (774M, <a href="#">Radford et al., 2019</a> )	1.093	
Llama-3.2-Instruct (1B)	0.807	
Llama-3.2-Instruct (3B)	0.737	
Gemma-2 (2B, <a href="#">Team et al., 2024</a> )	0.721	
Llama-3.2 (1B)	0.697	
Phi-3 (3.8B, <a href="#">Abdin et al., 2024</a> )	0.679	0.678
Phi-3 (7B, <a href="#">Abdin et al., 2024</a> )	0.678	
Gemma-2 (9B, <a href="#">Team et al., 2024</a> )	0.670	
GPT-3 (175B, <a href="#">Brown et al., 2020</a> )	0.666	
Phi-3 (14B, <a href="#">Abdin et al., 2024</a> )	0.651	
Llama-3.2 (3B)	0.640	
Gemma-2 (27B, <a href="#">Team et al., 2024</a> )	0.629	
<hr/>		
<i>Test-Time FT with SIFT + GPT-2 (124M)</i>	0.862	
<i>Test-Time FT with SIFT + GPT-2 (774M)</i>	0.762	
<i>Test-Time FT with SIFT + Phi-3 (3.8B)</i>	<b>0.595</b>	<b>0.599</b>

Table 2: Evaluation of state-of-the-art models on the Pile language modeling benchmark, without copyrighted datasets. Results with GPT-3 are from [Gao et al. \(2020\)](#). Results with Jurassic-1 and GLM are from [Zeng et al. \(2022\)](#) and do not report on the Wikipedia dataset. For a complete comparison, we also evaluate our Phi-3 with test-time fine-tuning when excluding the Wikipedia dataset.