

Università degli Studi di Padova



Exploiting causality methods for knowledge discovery from observational data

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Università degli Studi di Padova

Why study CAUSALITY?



Because

CORRELATION IS NOT CAUSATION



CORRELATION IS NOT CAUSATION





CORRELATION IS NOT CAUSATION

...but sometimes it gives strong hints





Because

CORRELATION IS NOT CAUSATION



Because

CORRELATION IS NOT CAUSATION

... and understanding how things work (causation) is the ultimate goal of science



... and understanding how things work (causation) is the ultimate goal of science

- 1. Which variables are "important" in my scenario?
- 2. How does changing one variable affect the system?



... and understanding how things work (causation) is the ultimate goal of science

1. Which variables are "important" in my scenario? \approx Feature selection

2. How does changing one variable affect the system?



Linear regression

$$\hat{y}_i = \mathbf{w}_0 + \mathbf{w}_1 \, x_1 + \dots + \mathbf{w}_N \, x_N$$

Trained by minimizing the mean squared error

$$\boldsymbol{w} = \underset{\boldsymbol{w}}{\operatorname{argmin}} \frac{1}{N} \sum_{i} (y_i - \hat{y}_i)^2$$



Linear regression

$$\hat{y}_i = \mathbf{w}_0 + \mathbf{w}_1 \, x_1 + \dots + \mathbf{w}_N x_N$$

LASSO focuses on w such that

$$\boldsymbol{w} = \underset{\boldsymbol{w}}{\operatorname{argmin}} \frac{1}{N} \sum_{i} (y_i - \hat{y}_i)^2 - \lambda \sum_{i} ||w_i||$$



Correlational approaches fail at discovering "important" variables

Solution obtained minimizing MSE

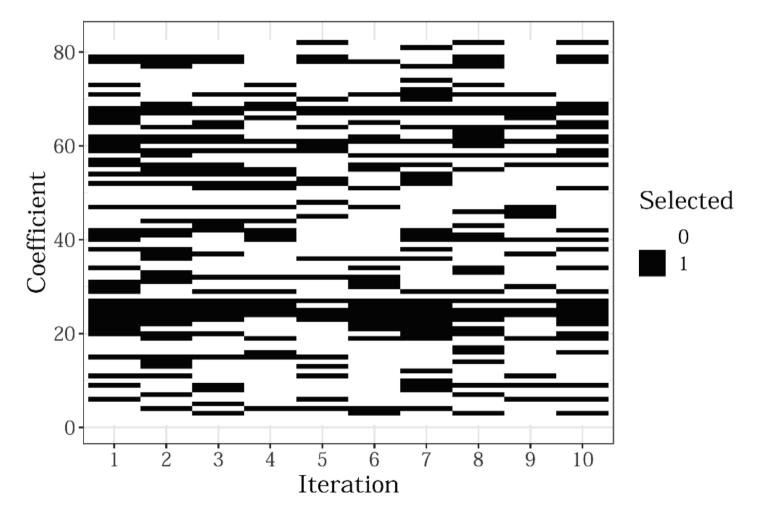
$$\hat{y}_i = w_0 + w_1 x_1 + w_2 x_2 + \dots + w_{N-1} x_{N-1} + w_N x_N$$

Solution obtained minimizing LASSO

$$\hat{y}_i = w_0 + w_1 x_1 + w_2 x_2 + \dots + w_{N-1} x_{N-1} + w_N x_N$$

UNIVERSITÀ DE COrrelational approaches fail at discovering "important" variables

FIGURE 2: INCLUDED PREDICTORS IN A LASSO REGRESSION ACROSS TEN SAMPLES FROM THE SAME POPULATION



Gillis, T.B. and Spiess, J.L., 2019. Big data and discrimination. The University of Chicago Law Review, 86(2), pp.459-488



... and understanding how things work (causation) is the ultimate goal of science

1. Which variables are "important" in my scenario? \approx Feature selection

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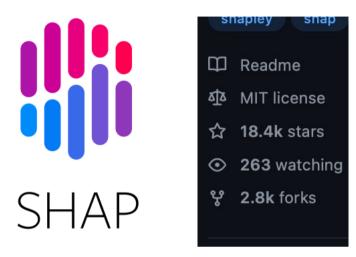
... and understanding how things work (causation) is the ultimate goal of science

- 1. Which variables are "important" in my scenario?
- 2. How does changing one variable affect the system? \approx Interpretation



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Correlational approaches fail at discovering effects



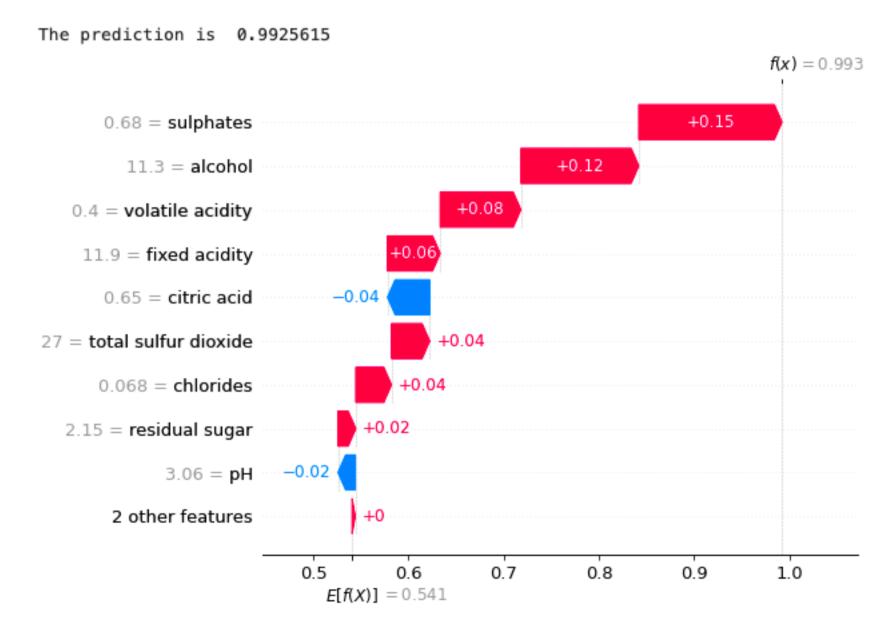
A unified approach to interpreting model predictions <u>SM Lundberg</u>, <u>SI Lee</u> - Advances in neural information ..., 2017 - proceedings.neurips.cc

Understanding why a model makes a certain prediction can be as crucial as the prediction's accuracy in many applications. However, the highest accuracy for large modern datasets is often achieved by complex models that even experts struggle to interpret, such as ensemble or deep learning models, creating a tension between accuracy and interpretability. In response, various methods have recently been proposed to help users interpret the predictions of complex models, but it is often unclear how these methods are related and ...

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[PDF] neurips.cc

Correlational approaches fail at discovering effects



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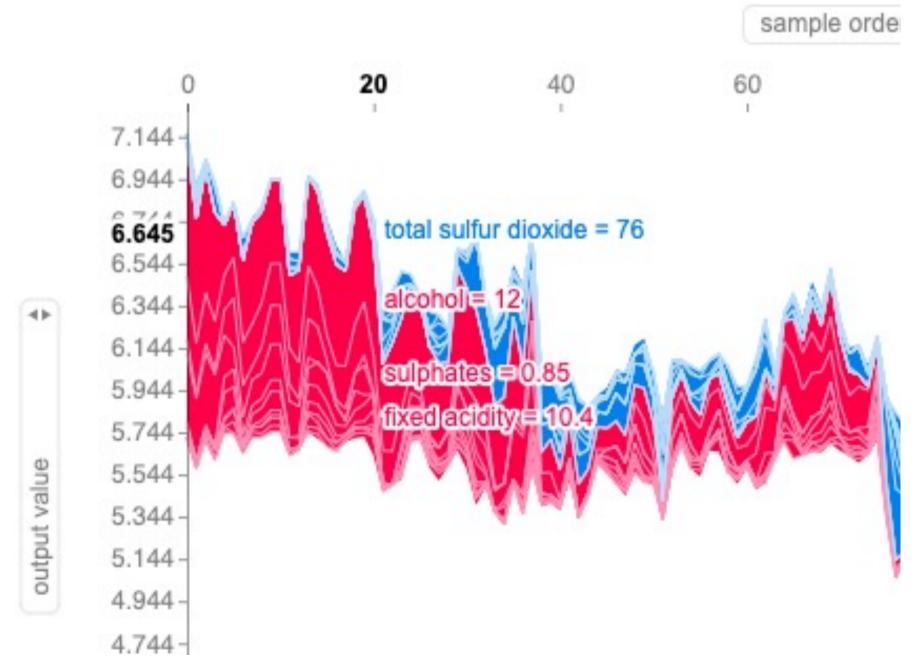


Correlational approaches fail at discovering effects



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Correlational approaches fail at discovering effects

```
[96] #!pip install shap
import shap
import numpy as np
import sklearn
```

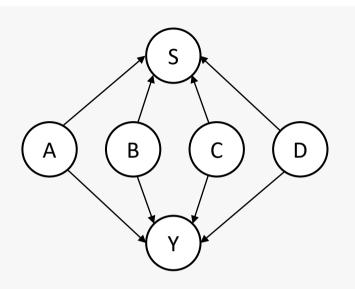
```
shap.initjs()
```

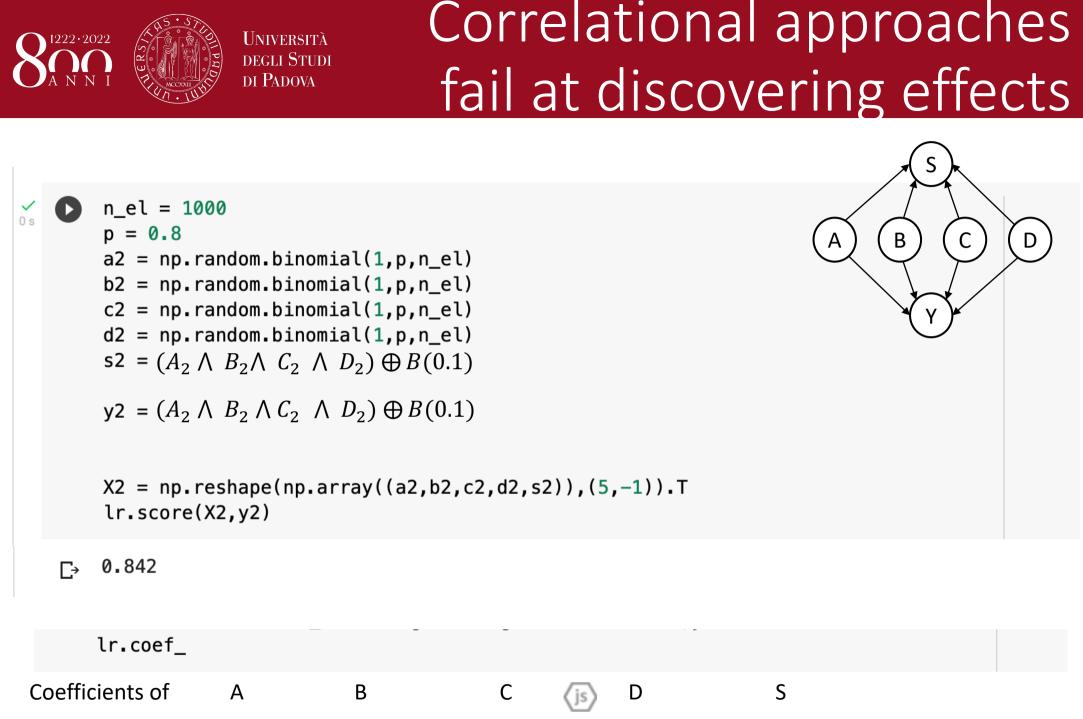
 $n_el = 250$ p = 0.8 $a = np.random.binomial(1,p,n_el)$ $b = np.random.binomial(1,p,n_el)$ $c = np.random.binomial(1,p,n_el)$ $d = np.random.binomial(1,p,n_el)$ $s = (A \land B \land C \land D) \oplus B(0.1)$

 $\mathsf{y} = (A \land B \land C \land D) \oplus B(0.1)$

X = np.reshape(np.array((a,b,c,d,s)),(5,-1)).T

lr = sklearn.linear_model.LogisticRegression().fit(X,y)





array([[1.11347288, 1.28590981, 0.98832866, 1.25790585, 2.09644493]])



Correlational approaches fail at discovering effects

✓ **[100]**

shap.initjs()

shap.force_plot(explainer.expected_value, shap_values, pd.DataFrame(X,columns=["A","B'





...and understanding how things work (causation) is the ultimate goal of science

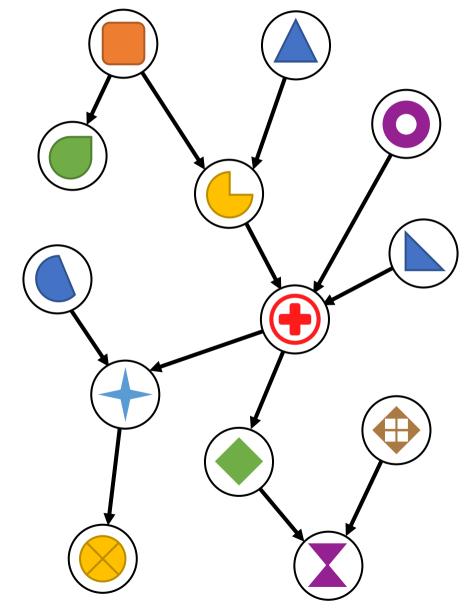
- 1. Which variables are "important" in my scenario?
- 2. How does changing one variable affect the system?



UNIVERSITÀ DEGLI STUDI DI PADOVA Causal framework (à la Pearl)

Causal Bayesian Networks are convient ways to model causal relationships among variables

 $BN = \langle G, p \rangle$ where $p(X_i) = f(pa(X_i), \epsilon)$ Interventional probability





(Some) main objectives of causal techniques:

Which variables are "important" in my scenario?
 Structure discovery: How are variables linked?



• How does changing one variable affect the system? **Effect estimation:** How changing () affects ()



Develop algorithms to:

- (Structure discovery) Discover causally related variables to a target
- (Effect estimation) Evaluate effect of causal rules

From <u>observational data</u> and providing guarantees on the results



Causal BNs are built using **interventional data** (e.g. setting variable $X_i = x_i$)

Observational data is much more common

Th. [informal] If spurious dependencies are removed, observational and interventional probability distributions are equivalent



Develop algorithms to:

- (Structure discovery) Discover causally related variables to a target
- (Effect estimation) Evaluate effect of causal rules

From observational data and <u>providing guarantees</u> on the results



Statistical guarantees are NOT just evaluation of perfomances

Guarantees are fundamental to gain users trust

Typically, algorithms with guarantees focus on False Discoveries (or False Positives)



Develop algorithms to:

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From <u>observational data</u> and providing <u>guarantees</u> on the results



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Develop algorithms to:

• (Structure discovery) Discover causally related variables to a target

Simionato Dario, and Fabio Vandin. "Bounding the Family-Wise Error Rate in Local Causal Discovery using Rademacher Averages."

Accepted at ECML PKDD 2022 – Best paper award



Task: Given a dataset of observations of variables *V*, find those **causally** related to *T* with **guarantees** on the result (e.g. no false positives)

Useful in: Biology, medicine, neuroscience

Set of variables V							Target 2	
		\bigoplus			\bigcirc			

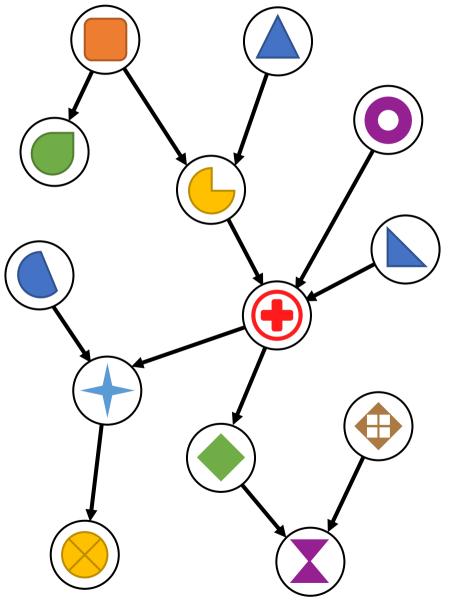


Causal Bayesian Networks represent cause-consequence relations between variables

Informally, if () is a **cause** of (),

then fixing all variables values and changing

the value of leads to a variation on the values of



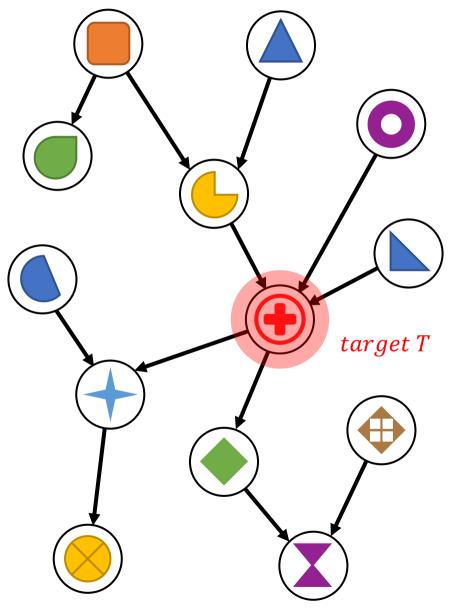


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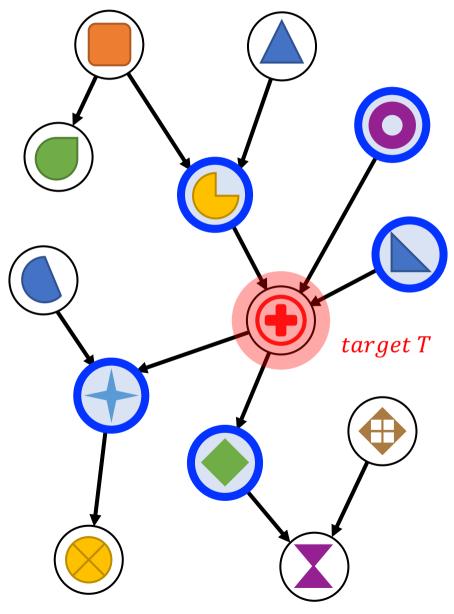




Local causal discovery focuses on:

• Parent – Children set of T PC(T)Parents(T) + Children(T)

Contains direct causes and consequences of *T*





Local causal discovery focuses on:

• Parent – Children set of T PC(T)Parents(T) + Children(T)

Contains direct causes and consequences of *T*

• Markov Boundary of $T \not = MB(T)$ PC(T) + Spouses(T)

Optimal set for the prediction of *T*

target T



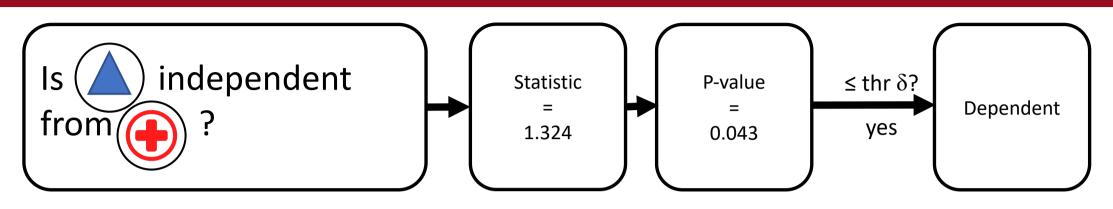
Problem Definition

Given a dataset and a target *T*, our task is to discover:

• Parent – Children set of T PC(T)• (or) Markov Boundary of T MB(T)• (or) Markov Eoundary of T MB(T)

Several approaches are proposed in the literature [Pena et al. '07], [Aliferis et al. '10], [Aliferis et al. '03]

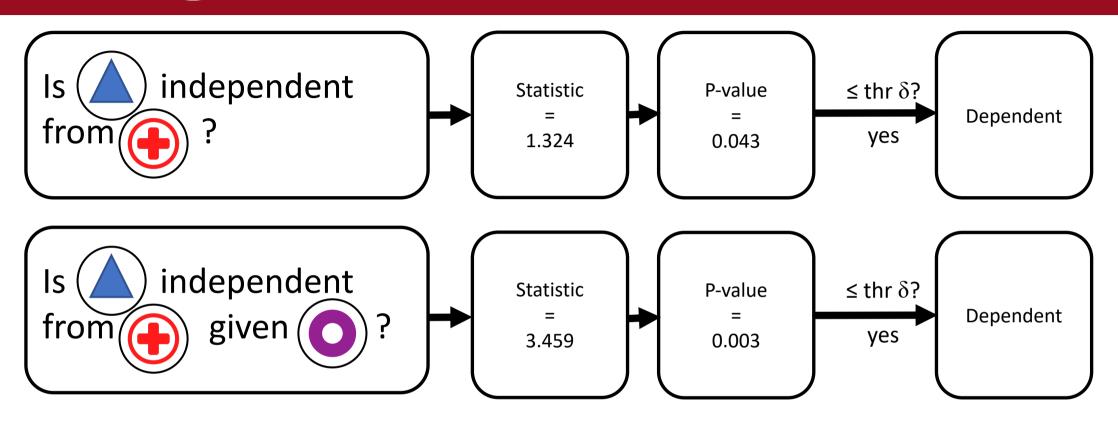






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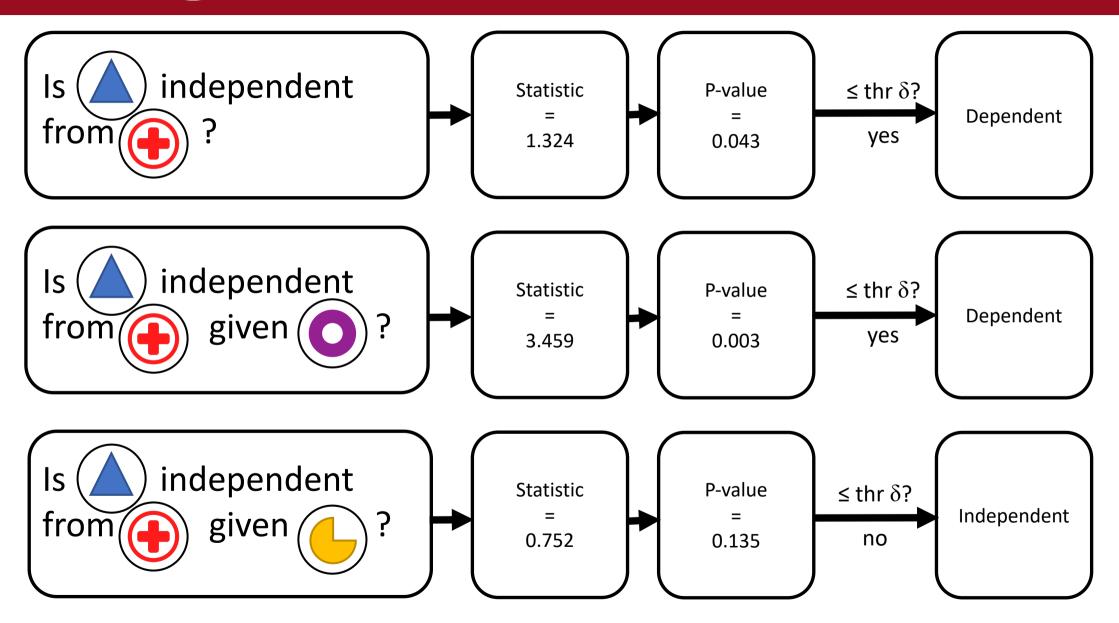
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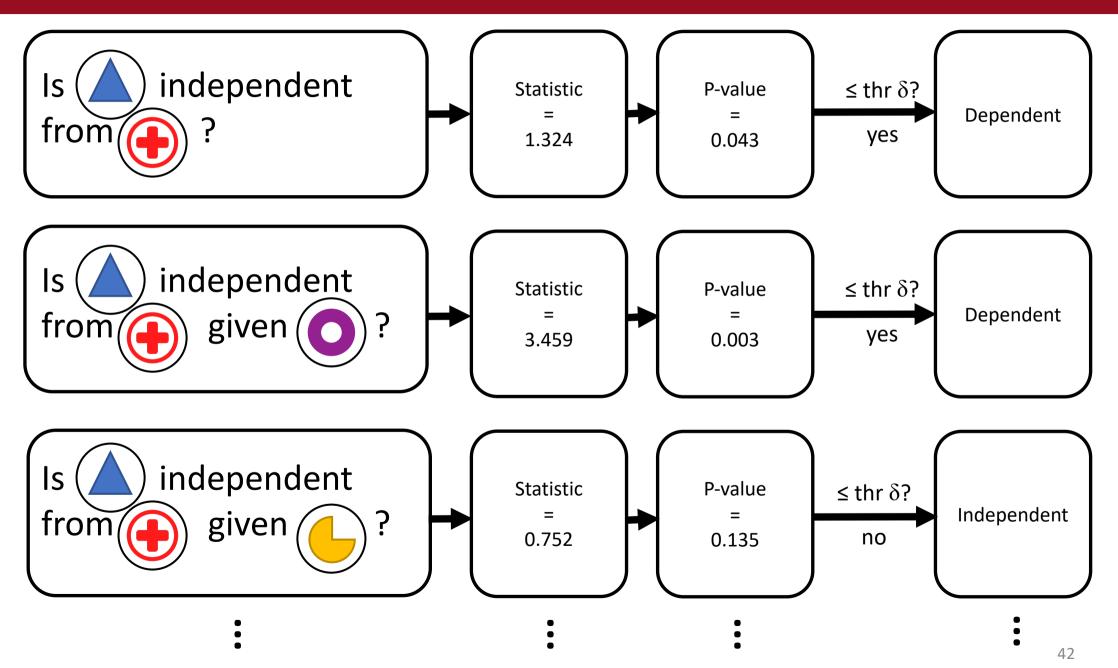
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When testing *N* hypotheses

Number of false positives ~ $Bin(N, \delta)$

 $P(At \ least \ one \ false \ positive) = 1 \ - (1 - \delta)^N \ge \delta$

In our problem $N = |V|(|V| + 1)2^{|V|-2}$ is the number of possible conditional independence tests.



Statistical Testing and Guarantees

Approaches with guarantees typically focus on bounding

Correct solution



False Discovery Rate

$$FDR = E\left[\frac{Number\ false\ discoveries}{Total\ discoveries}\right]$$

Typical solution (controlling the FDR)



Family-Wise Error Rate

FWER = P(Return at least one false positive) Typical solution (controlling the FWER)





Statistical Testing and Guarantees

Approaches with guarantees typically focus on bounding

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False Discovery Rate

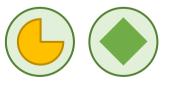
$$FDR = E\left[\frac{Number \ false \ discoveries}{Total \ discoveries}\right]$$

Typical solution (controlling the FDR)



Family-Wise Error Rate

FWER = P(Return at least one false positive) Typical solution (controlling the FWER)







Assume perfect detection of dependencies and independencies

[Pena et al. '07], [Aliferis et al. '10], [Aliferis et al. '03]

- Unfeasible and untestable assumptions

Bound the false discovery rate

[Tsamardinos and Brown '08]

- May still return false positives



- Proved that **SoA algorithms cannot control the** *FWER* by correcting for multiple hypothesis testing
- Developed RAveL-PC and RAveL-MB: the first algorithms for local causal discovery with guarantees on the FWER
- Implemented bounds on FWER exploiting classical corrections and data-dependent bounds based on Rademacher averages
- Tested RAveL-PC and RAveL-MB both on synthetic and real-world datasets



Th. [informal] State of the art algorithms (GetPC, PCMB and IAMB) control the *FWER* if they correct for multiple hypothesis testing and some strong assumptions (infinite power) are met.

We showed some examples in which **removing such assumptions** may lead to returning **false positives in output**



RAveL Pipeline

Our algorithms:

- Formulate the local discovery task using only independence tests (and not dependence tests)
- Apply suitable corrections (Rademacher, Bonferroni) to control for the *FWER*



RAveL Pipeline

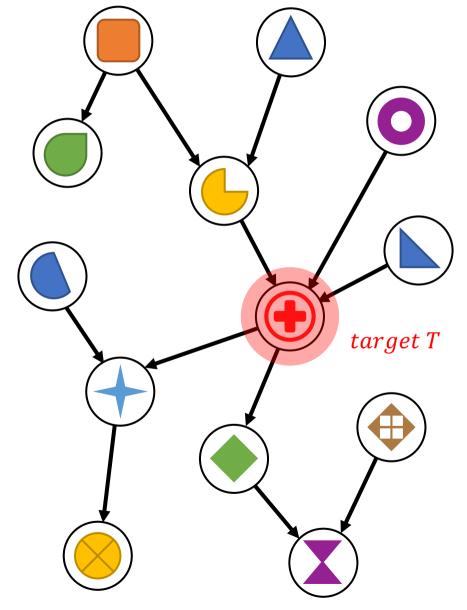
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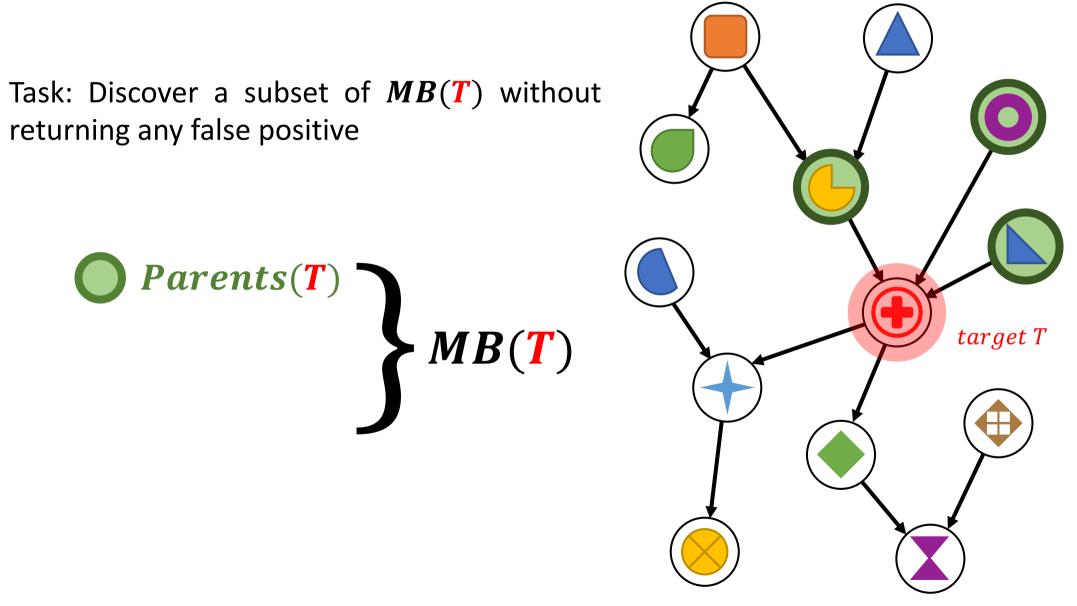
Th. [informal] RAveL-PC and RAveL-MB effectively control the FWER below a given threshold δ .



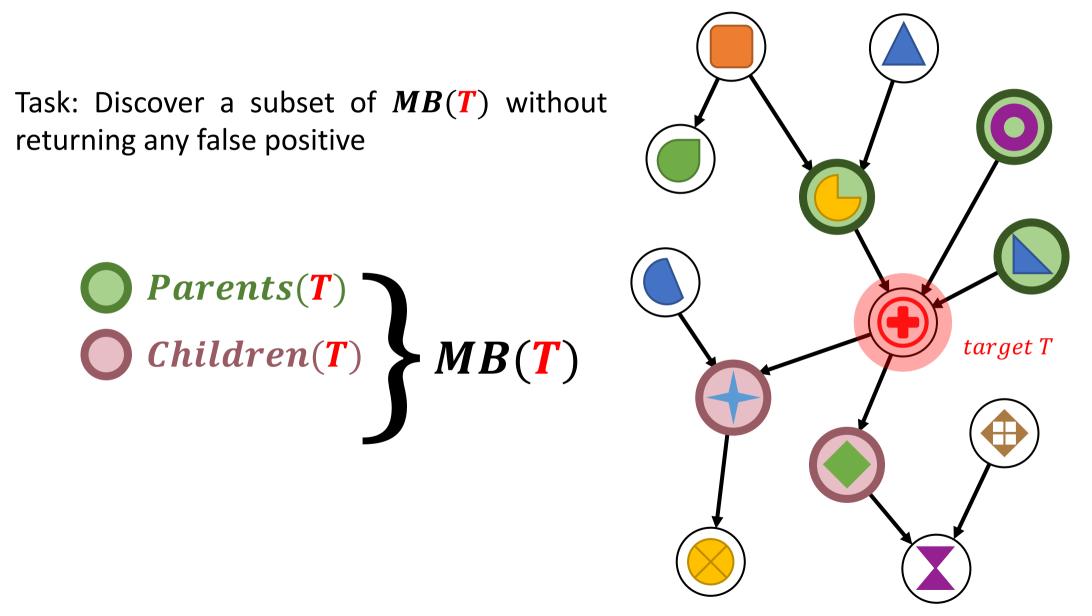
Task: Discover a subset of MB(T) without returning any false positive







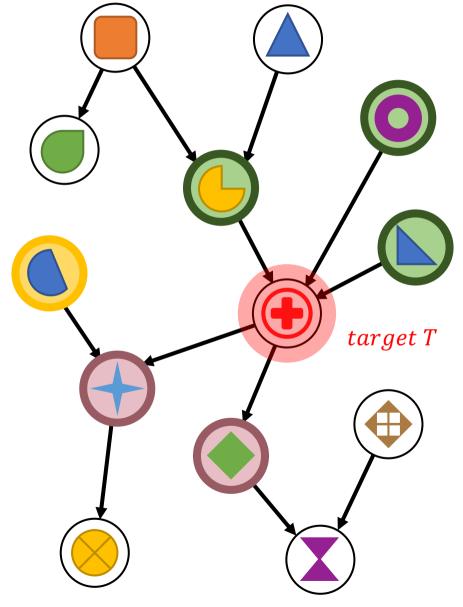






Task: Discover a subset of MB(T) without returning any false positive

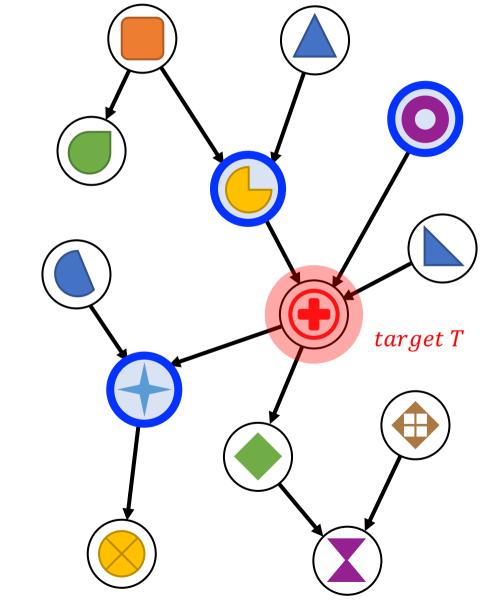
Parents(T)
Children(T)
Spouses(T)





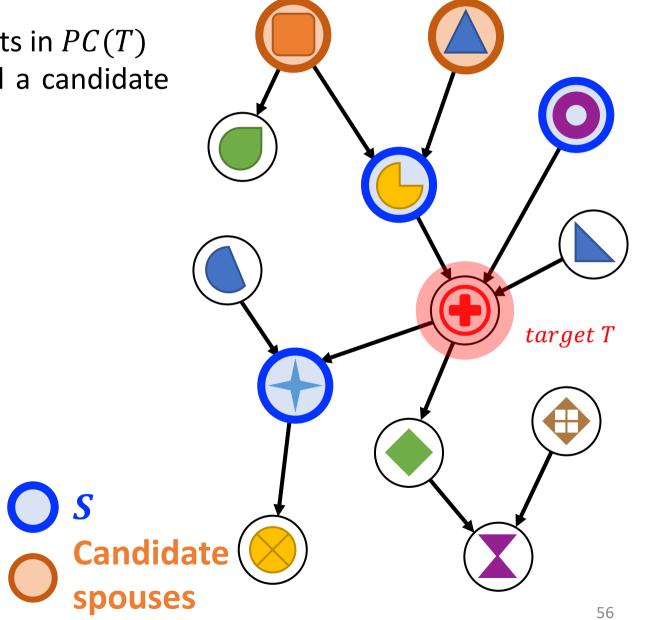
1- Discover a subset **S** of elements in PC(T)

 $\bigcirc S$



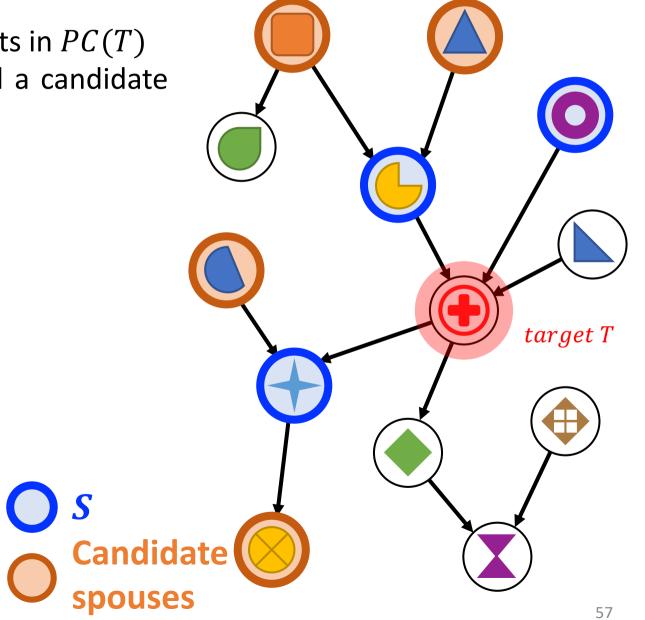


1- Discover a subset **S** of elements in PC(T)2- For each element X in S find a candidate set of spouses in PC(X)





1- Discover a subset S of elements in PC(T)
2- For each element X in S find a candidate set of spouses in PC(X)





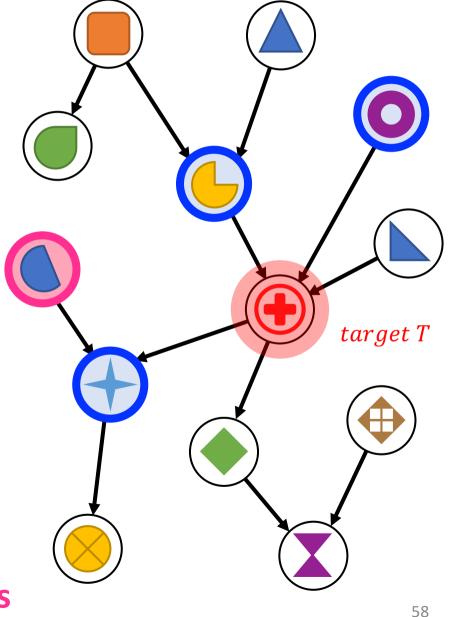
1- Discover a subset **S** of elements in PC(T)2- For each element X in S find a candidate set of spouses in PC(X)

3- For each candidate spouse Y with children

X, test the spouse condition

Previous approaches:

Independence test Dependence test $Y \perp T | \mathbf{Z} \text{ and } Y \perp T | \mathbf{Z} \cup \{X\}$ for all $\mathbf{Z} \subseteq \mathbf{V} \setminus \{X, Y, T\}$ Our formulation (equivalent): $\overline{Y \not \perp T | \mathbf{V} \setminus \{Y\}}$ **Actual**





1- Discover a subset **S** of elements in PC(T)2- For each element X in S find a candidate set of spouses in PC(X)3- For each candidate spouse Y with children

X, test the spouse condition

Previous approaches:

Dependence test Independence test target T $Y \perp T \mid \mathbf{Z} \text{ and } Y \perp T \mid \mathbf{Z} \cup \{X\}$ for all $\mathbf{Z} \subseteq \mathbf{V} \setminus \{X, Y, T\}$ Our formulation (equivalent): $\overline{Y \not \perp T} | \mathbf{V} \setminus \{Y\}$ **Actual** 59



- Bonferroni correction on the threshold δ

Classical correction for multiple hypotheses testing uses a modified threshold δ/N on each test

May be too strict if N is big (and in our case N is exponential on |V|)

• Rademacher averages to bound each test statistic Provide data-dependent bounds



Given a family of functions \mathcal{F} and a dataset S, Rademacher averages upper bound with high probability the Supremum Deviation $D(\mathcal{F}, S)$

Empirical sample meanRademacher
estimate $D(\mathcal{F}, S) = \sup_{f \in \mathcal{F}} \left| \hat{E}_S[f] - E[f] \right| \le 2\tilde{R} + O(\frac{1}{\sqrt{m}})$ ExpectationDataset
size



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Empirical sample meanRademacher
estimate $D(\mathcal{F}, S) = \sup_{f \in \mathcal{F}} \left| \hat{E}_S[f] - E[f] \right| \le 2\tilde{R} + O(\frac{1}{\sqrt{m}})$ ExpectationDataset
size

They can lower bound **simultaneously** each independence test statistic for providing guarantees on the *FWER*



Rademacher Averages - Idea

Given two normalized vectors of observations x, y

Pearson's r coefficient

$$r_{x,y} = \frac{\sum_{i=0}^{m-1} x_i y_i}{(m-1)}$$



Given two normalized vectors of observations x, y

Pearson's r coefficient
$$r_{x,y} = \frac{\sum_{i=0}^{m-1} x_i y_i}{(m-1)}$$

By defining

$$r_{x,y}(s_i) = \frac{m}{(m-1)} x_i y_i$$

We get

$$r_{x,y} = \widehat{\mathbb{E}}_{S}[r_{x,y}(s_{i})]$$



Given two normalized vectors of observations x, y

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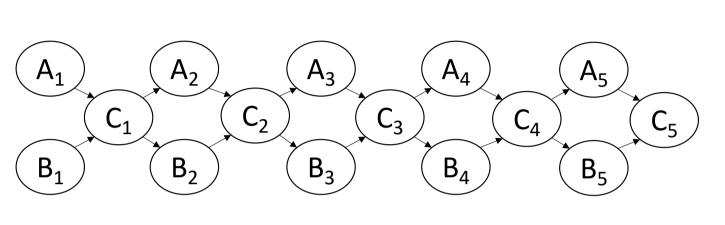
$$\mathcal{F} \ni r_{x,y}(s_i) = \frac{m}{(m-1)} x_i y_i$$

We get

$$r_{x,y} = \widehat{\mathbb{E}}_{S}[r_{x,y}(s_{i})]$$

The same approach works with other test statistics

Experimental Evaluation



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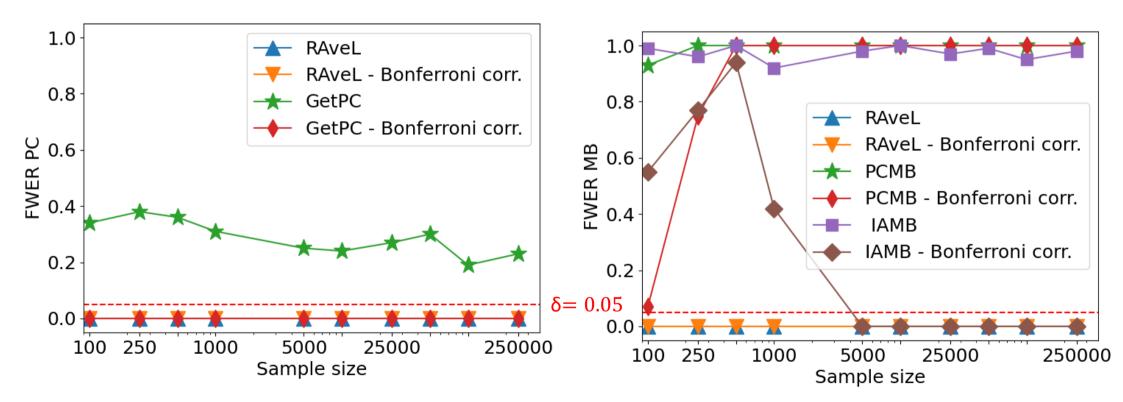
 $A_{1} \sim N(0, 1)$ $B_{1} \sim N(0, 1)$ $C_{1} \sim -3 + 2A_{1} + 3B_{1} + N(0, 1)$ $\forall i \in [2, 3, 4, 5]$ $A_{i} \sim +3 + 3C_{i-1} + N(0, 1)$ $B_{i} \sim -1 - 2C_{i-1} + N(0, 1)$ $C_{i} \sim -3 + 2A_{i} + 3B_{i} + N(0, 1)$

- On each run, test the SoA and RAveL algorithms on <u>every</u> variable
- Analyse results of each iteration:
 - No False Positives only if all the outputs (one per variable) of the run do not contain false positives
 - Count mean percentage of false negatives



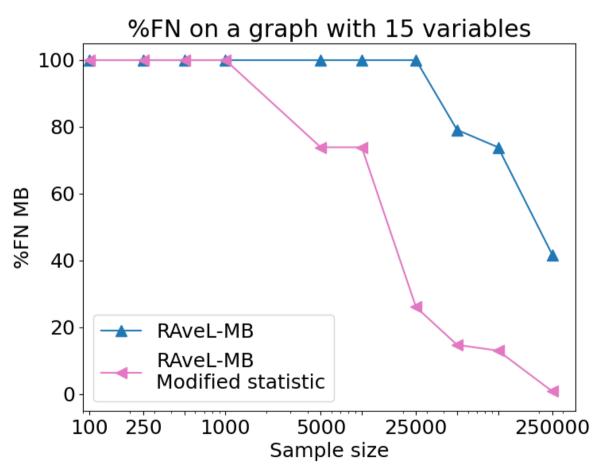
Experimental Evaluation

RAveL-PC and RAveL-MB effectively control the FWER





Tests with increasing number of variables

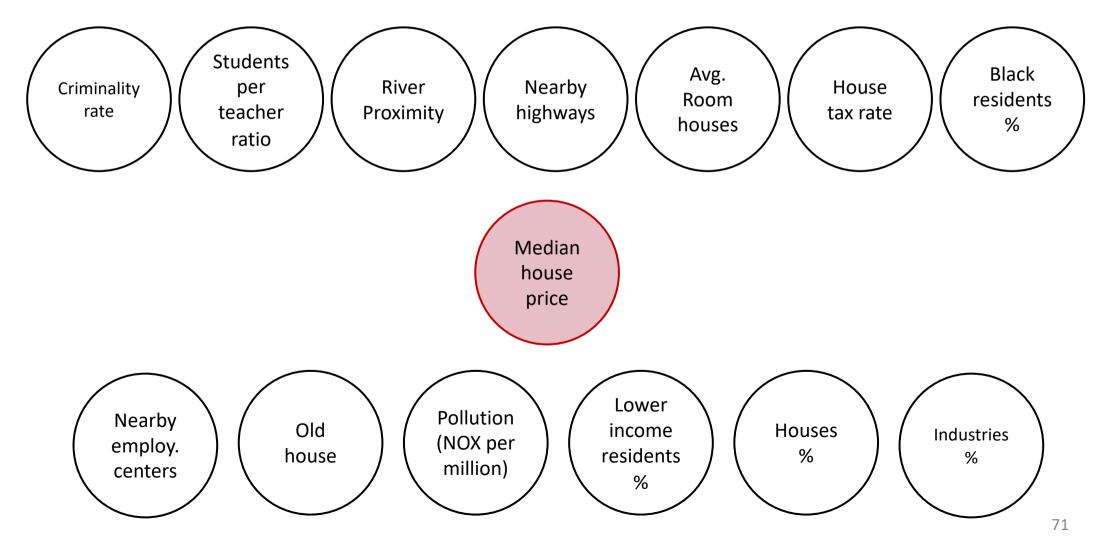


Test statistic choice heavily affects Rademacher based corrections



Experimental Evaluation

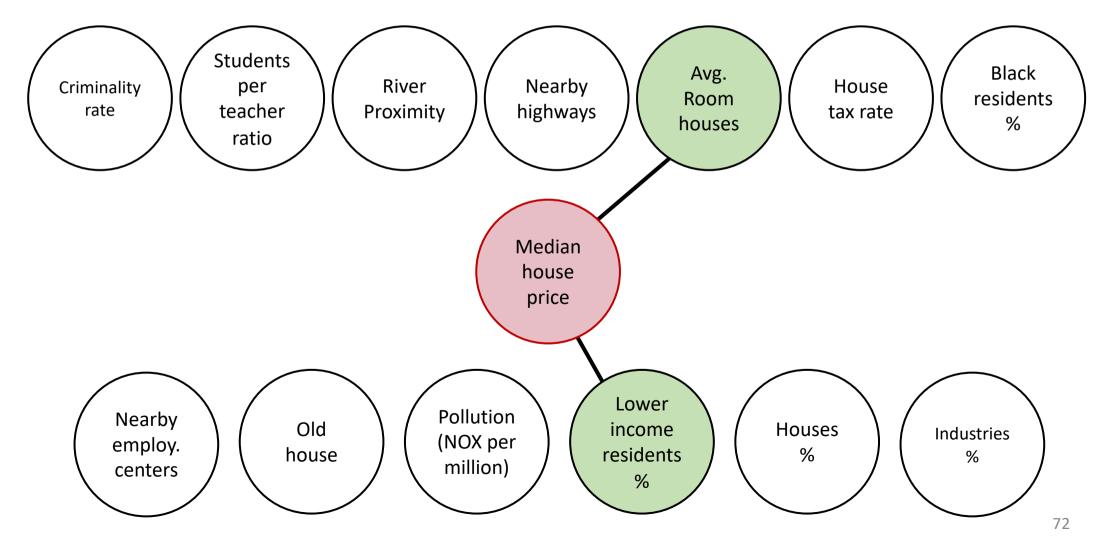
Tested RAveL-PC and RAveL-MB on **Boston housing** dataset with $\delta = 0,01$.





Experimental Evaluation

Tested RAveL-PC and RAveL-MB on **Boston housing** dataset with $\delta = 0,01$.





My focus

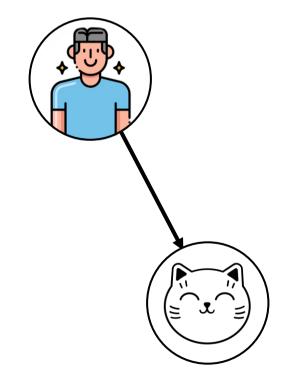
Develop algorithms to:

- (Structure discovery) Discover causally related variables to a target
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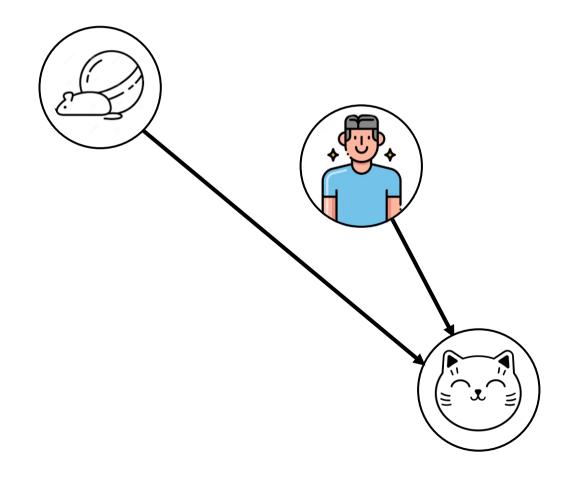
From <u>observational data</u> and providing <u>guarantees</u> on the results



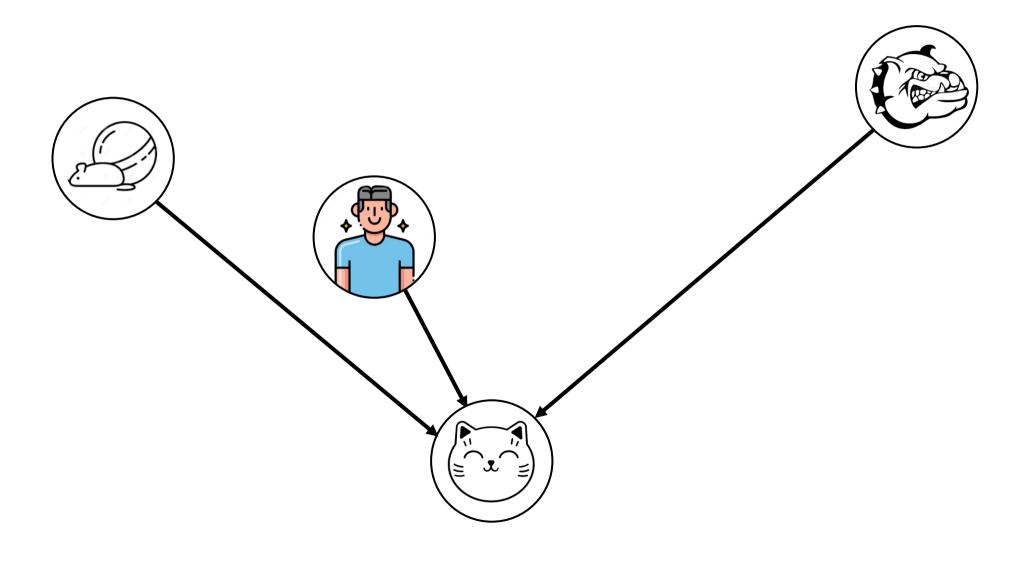
Very serious research question



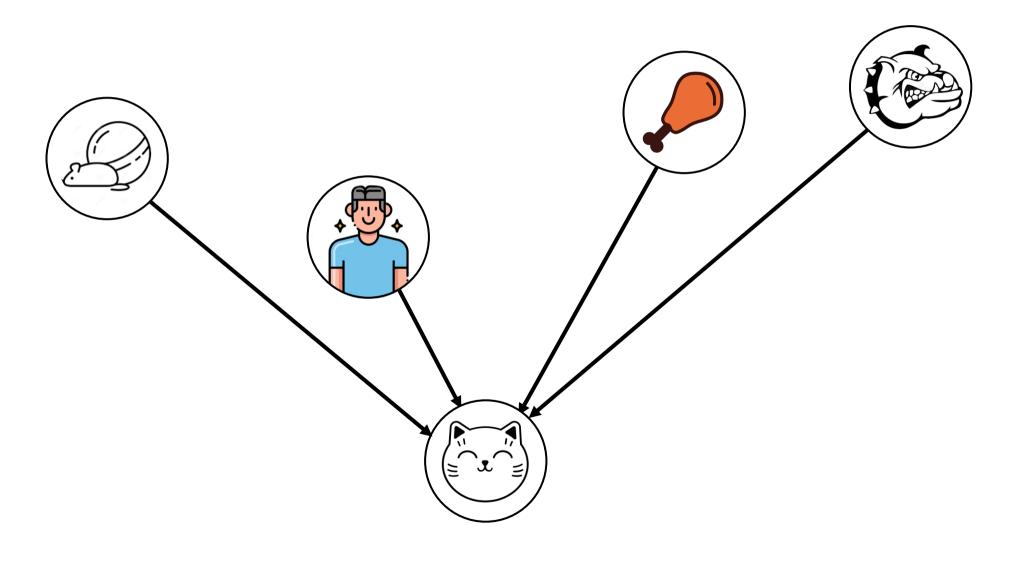




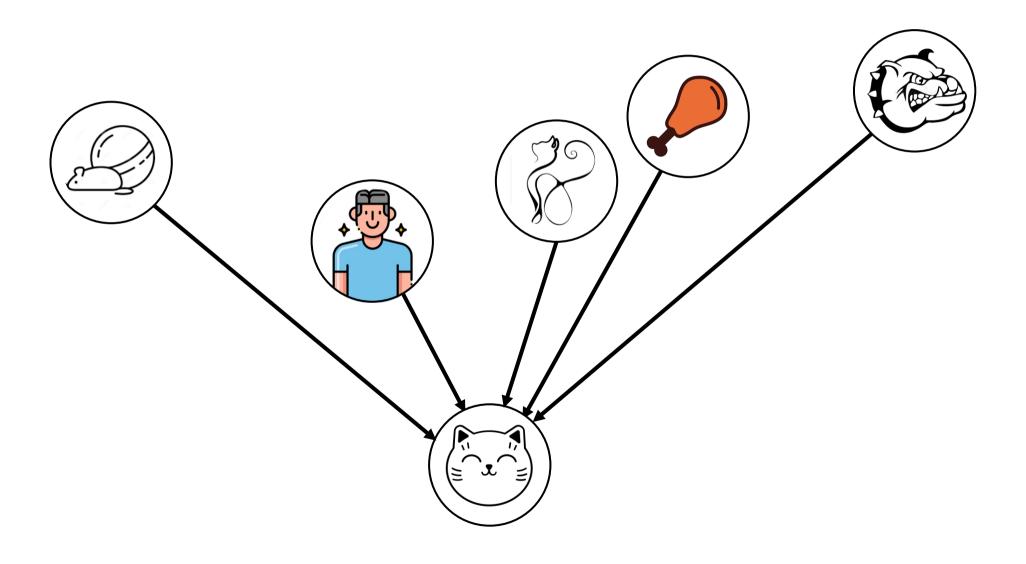




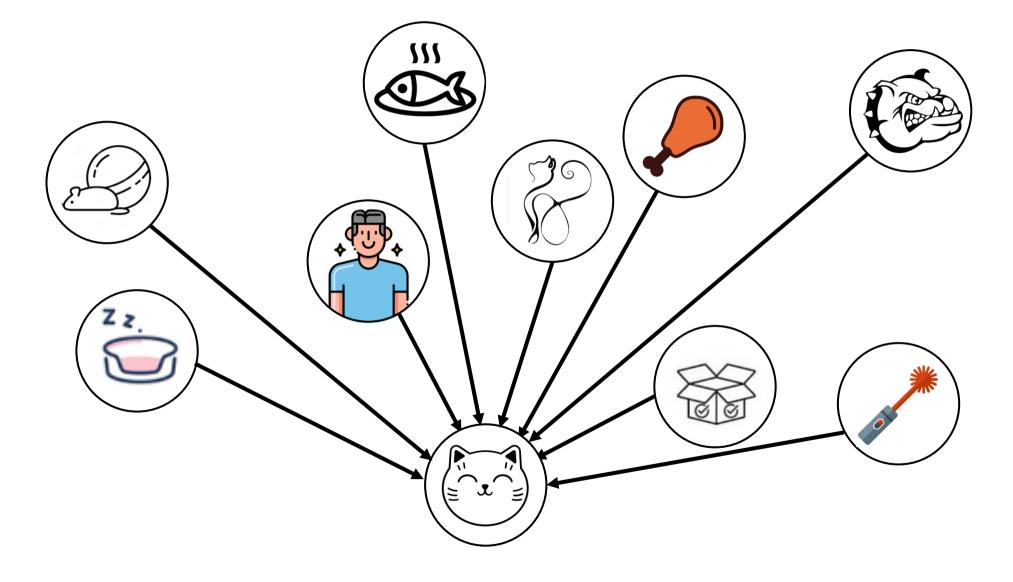














My focus

Develop algorithms to:

• (Structure discovery) Discover causally related variables to a target

• (Effect estimation) Evaluate effect of causal rules

Submitted paper at ECCB 2023 and work in progress in collaboration with:

Antonio Collesei, PhD s. (IOV)

Paola Donolato, MS

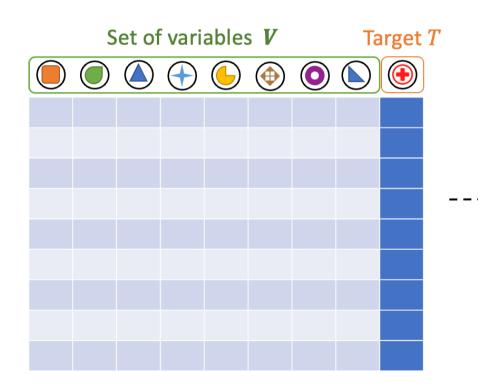
Federica Miglietta, MD and PhD s. (IOV)

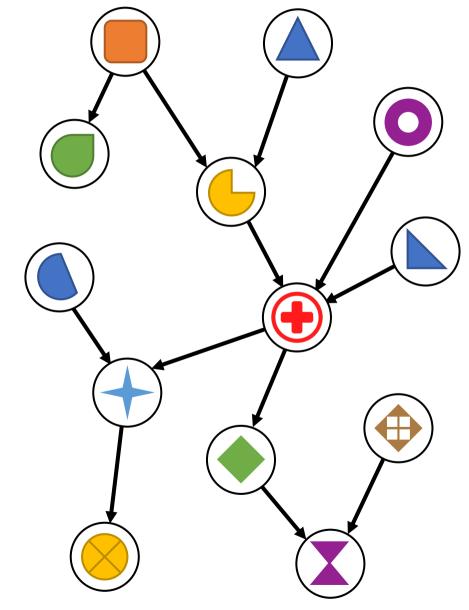
Fabio Vandin, Full Professor



Problem Setup

Causal Bayesian Networks represent causeconsequence relations between variables

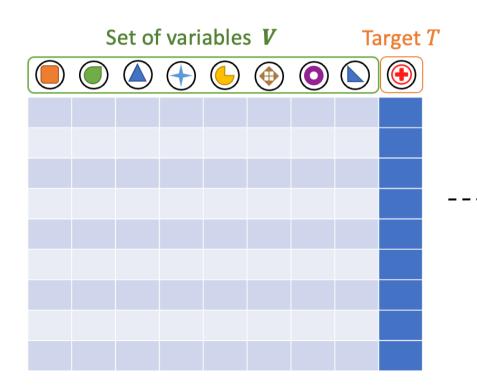


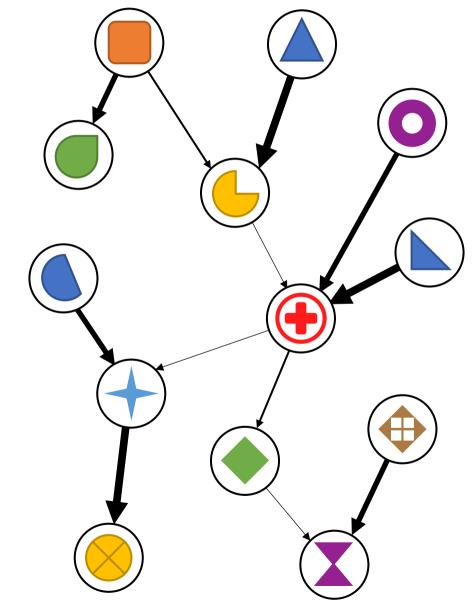




Problem Setup

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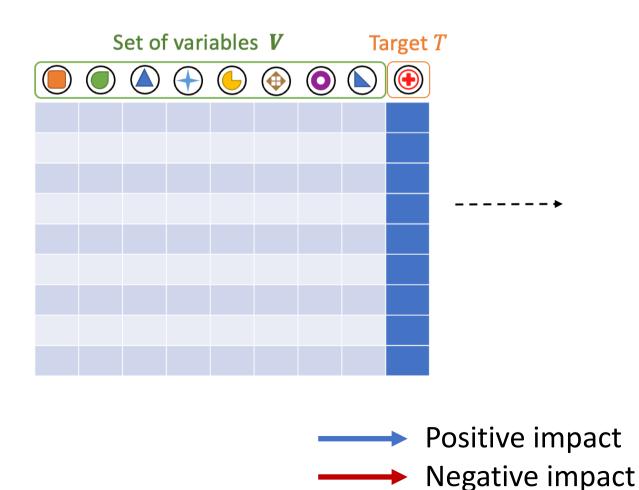


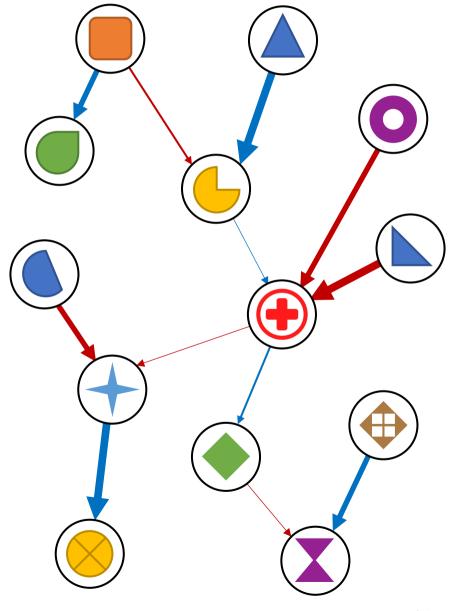




Problem Setup

Causal Bayesian Networks represent causeconsequence relations between variables







Problem Definition

$$\sigma: \bigcirc = \mathsf{TRUE} \text{ AND } \bigcirc = \mathsf{FALSE} \text{ THEN } \bigcirc = \mathsf{TRUE} \text{ GIVEN} \bigcirc \mathsf{GIVEN} \odot \mathsf{GIVEN} \bigcirc \mathsf{GIVEN} \bigcirc \mathsf{GIVEN} \bigcirc \mathsf{GIVEN} \bigcirc \mathsf{GIVEN} \bigcirc \mathsf{GI$$



Problem Definition

$$\sigma: \bigcirc = \mathsf{TRUE} \text{ AND } \bigcirc = \mathsf{FALSE} \text{ THEN } \bigcirc = \mathsf{TRUE} \text{ GIVEN } \bigcirc \mathsf{GIVEN} \odot \mathsf{GIVEN} \bigcirc \mathsf{GIVEN} \bigcirc \mathsf{GIVEN} \bigcirc \mathsf{GIVEN} \odot \mathsf{G$$

$$e(\sigma) = p(\textcircled{\textcircled{}} = TRUE | \sigma, \textcircled{}) - p(\textcircled{\textcircled{}} = TRUE | \overline{\sigma}, \textcircled{})$$



Problem Definition

$$\sigma: \bigcirc = \mathsf{TRUE} \text{ AND } \bigcirc = \mathsf{FALSE} \text{ THEN } \bigcirc = \mathsf{TRUE} \text{ GIVEN } \bigcirc \mathsf{GIVEN} \odot \mathsf{GIVEN} \odot \mathsf{GIVEN} \odot \mathsf{GIVEN} \odot \mathsf{GIVEN} \bigcirc \mathsf{GIVEN} \odot \mathsf{G$$

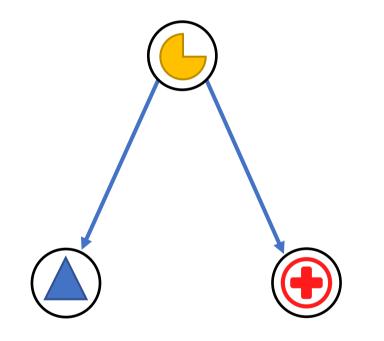
$$e(\sigma) = p\left(\bigoplus = TRUE | \sigma, \bigoplus \right) - p\left(\bigoplus = TRUE | \overline{\sigma}, \bigoplus \right)$$

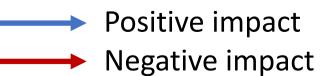
Task: Given a dataset of observations of variables V, find the top-k rules $\sigma_1^*, \ldots, \sigma_k^*$ with the highest causal effect on T with guarantees on the result (e.g. no false positives)





We have to deal with confounder variables

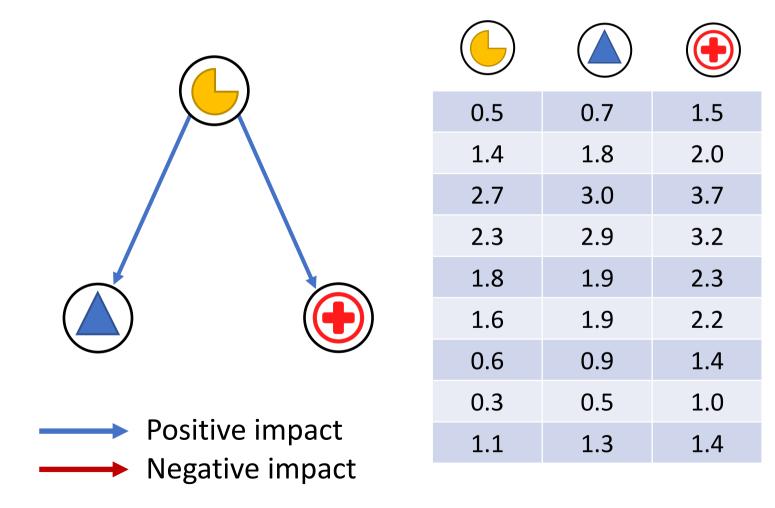






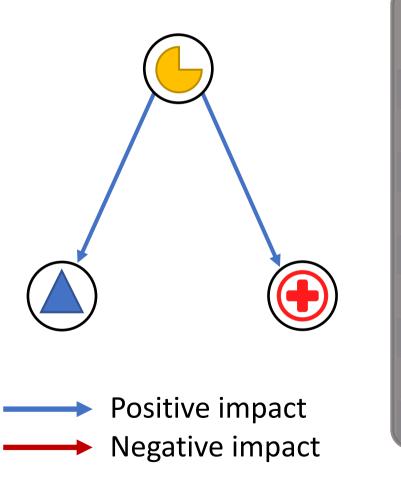
Example

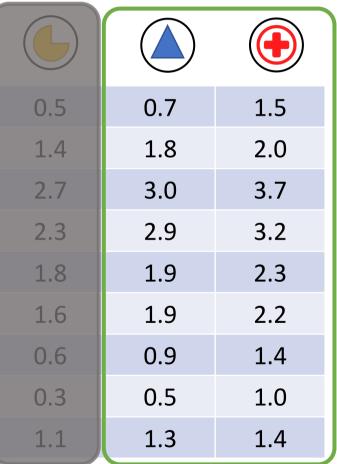
We have to deal with confounder variables

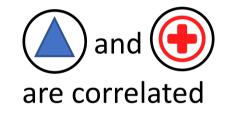




We have to deal with confounder variables



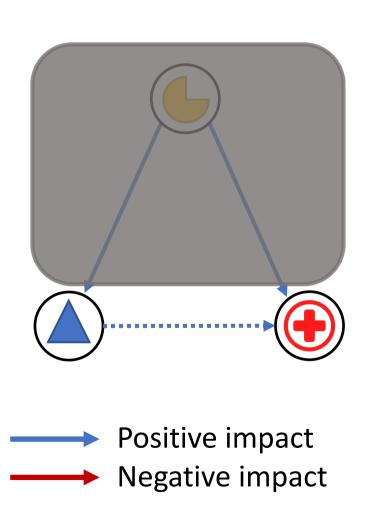




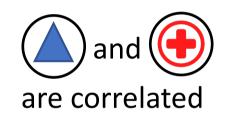
Example



We have to deal with confounder variables



0.5	0.7	1.5
1.4	1.8	2.0
2.7	3.0	3.7
2.3	2.9	3.2
1.8	1.9	2.3
1.6	1.9	2.2
0.6	0.9	1.4
0.3	0.5	1.0
1.1	1.3	1.4

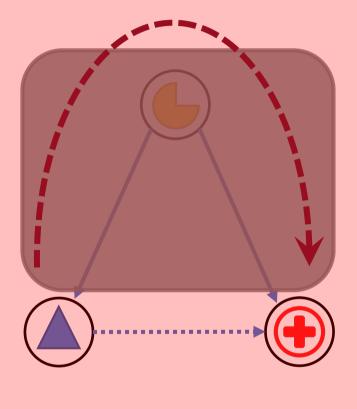


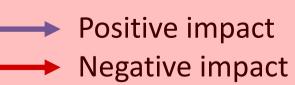
Example



Example

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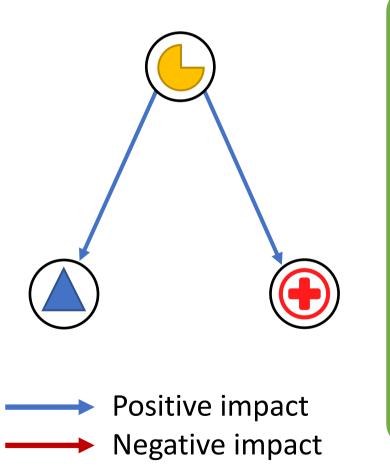


CORRELATION IS NOT CAUSATION

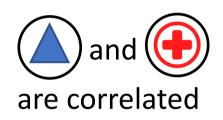


Example

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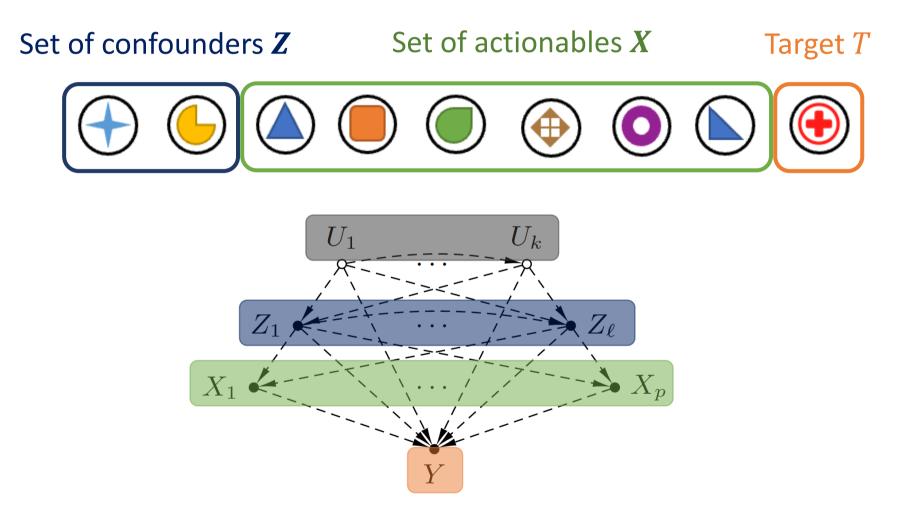
and are **NOT** correlated by conditioning on





Real-world data example

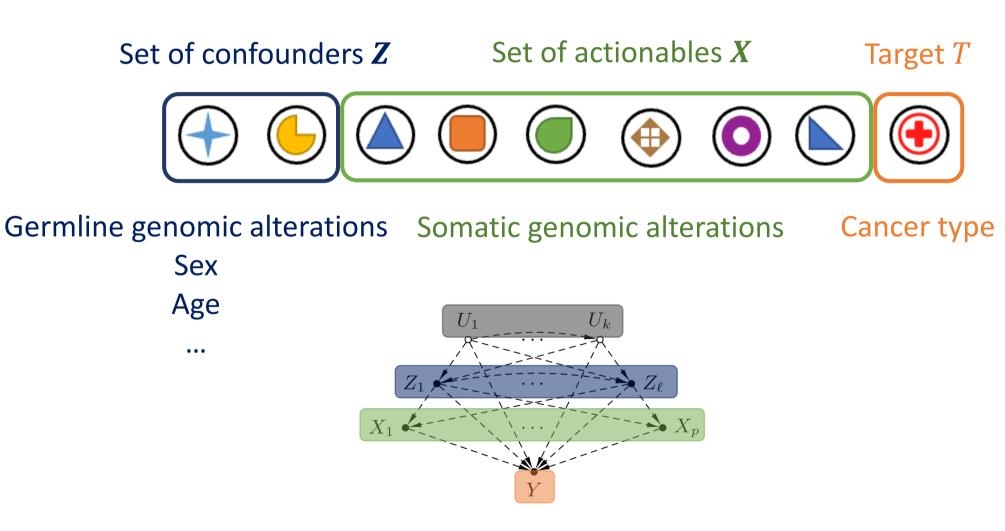
We are currently working with Breast Cancer Data





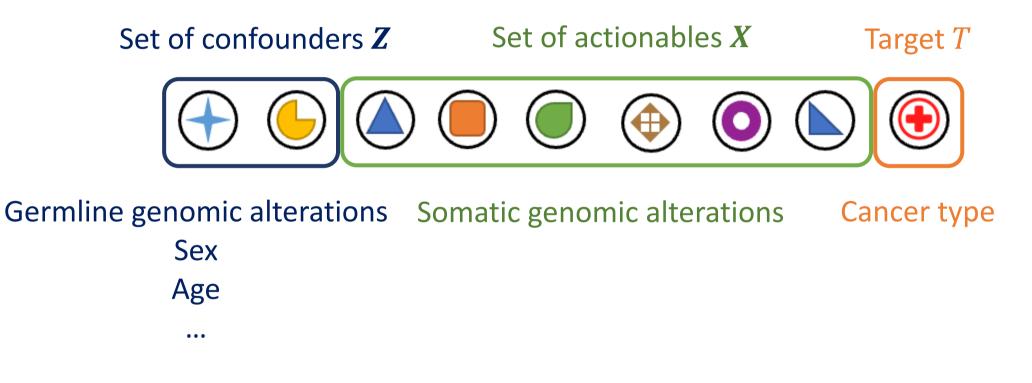
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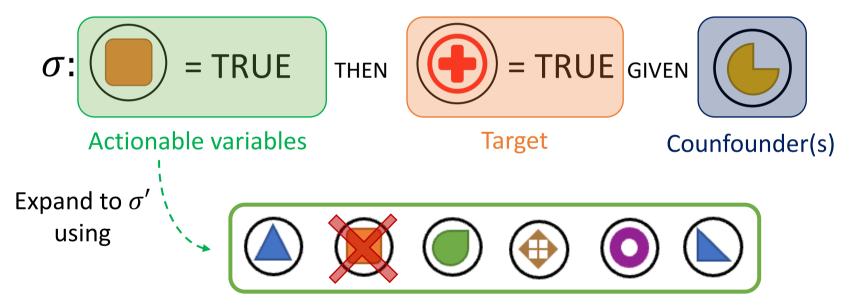
We are currently working with Breast Cancer Data



 $TP53_{som} = 1 \land ERBB2_{loh} = 1 \rightarrow Basal \mid history other malignancy$



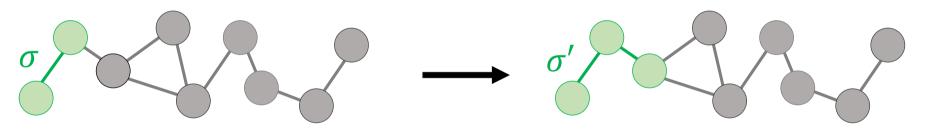
[Budhathoki et al.] proposed the reliable rule effect estimation framework and developed a branch and bound algorithm for the discovery task.



Budhathoki, K., Boley, M. and Vreeken, J., 2021. Discovering reliable causal rules. In *Proceedings of the 2021 SIAM International Conference on Data Mining (SDM)* (pp. 1-9). Society for Industrial and Applied Mathematics.



- Guarantees for multiple hypothesis testing
- Prove that the general problem is NP-hard
- Exploit dependency graph G for rule expansion



- Extensive experiments on breast cancer data
- Correction for multiple hypothesis testing issues Currently working on data dependent corrections with Rademacher Averages



My focus

Develop algorithms to:

- (Structure discovery) Discover causally related variables to a target
- (Effect estimation) Evaluate effect of causal rules

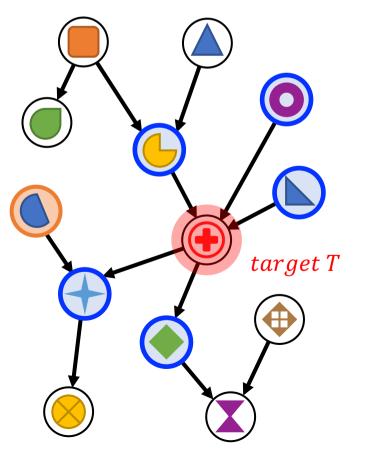
From <u>observational data</u> and providing <u>guarantees</u> on the results



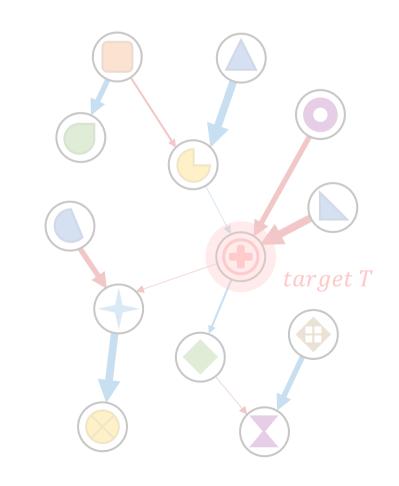
Short recap

1 - Structure discovery

Discover causally related variables to a target



2 - Effect estimation Evaluate effect of causal rules

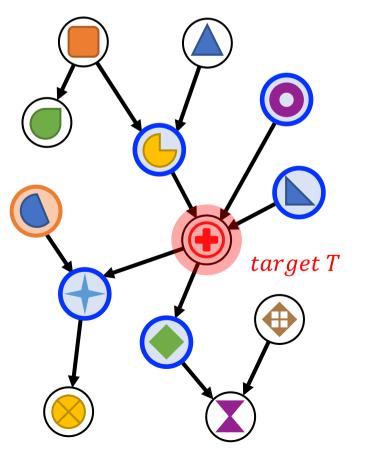




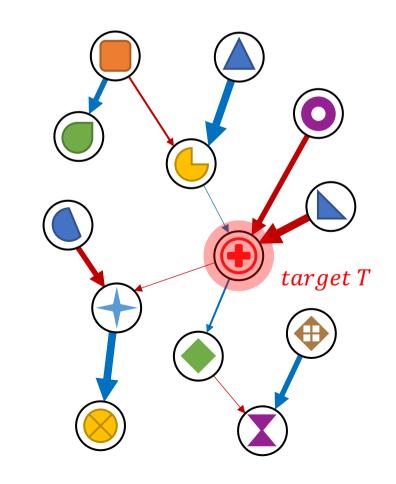
Short recap

1 - Structure discovery

Discover causally related variables to a target



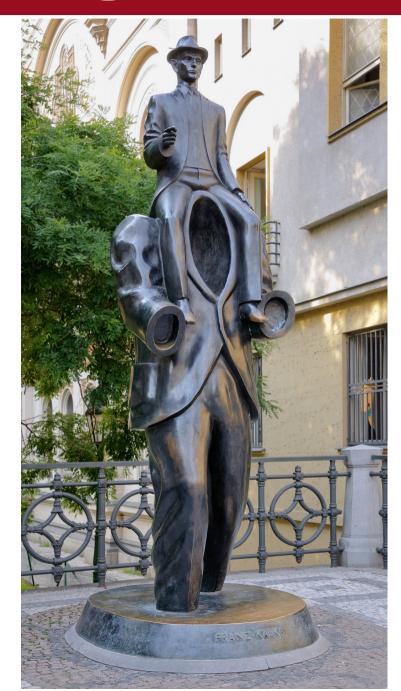
2 - Effect estimation Evaluate effect of causal rules



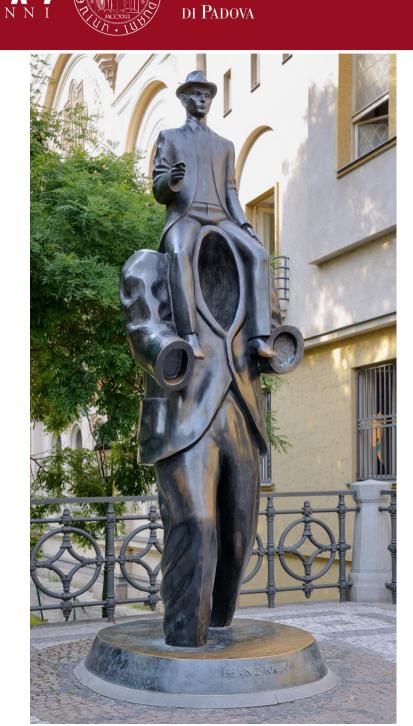




Hopefully true correlation example







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Exploiting causality methods for knowledge discovery from observational data

Q & A TIME

in Dario Simionato

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Padua, Italy

Code available at:

https://github.com/VandinLab/RAveL